Liquidity fragmentation on decentralized exchanges

Alfred Lehar[1], Christine Parlour[2], and Marius Zoican[3]

Haskayne School of Business @ University of Calgary

Haas School of Business @ UC Berkeley

Rotman School of Management @ University of Toronto

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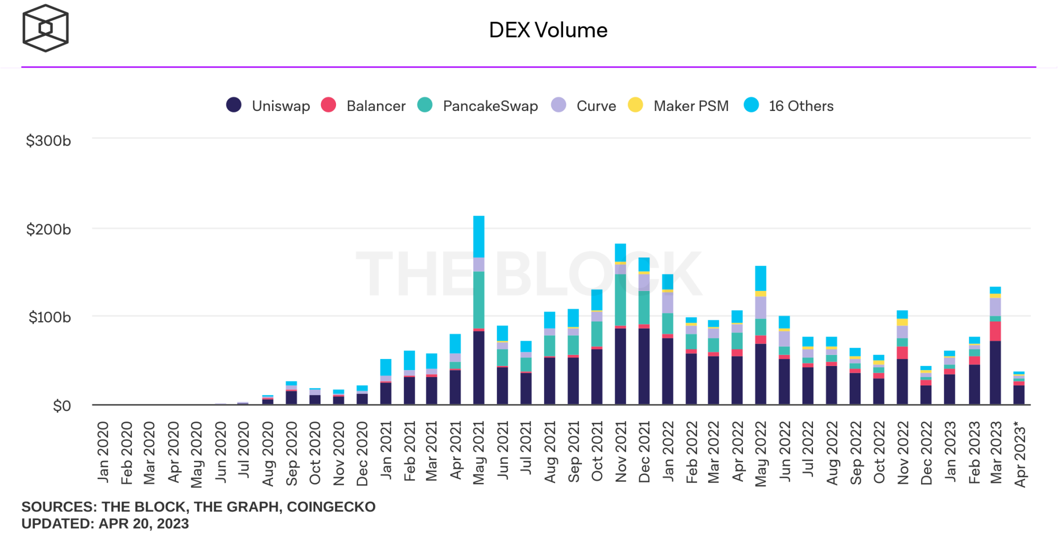
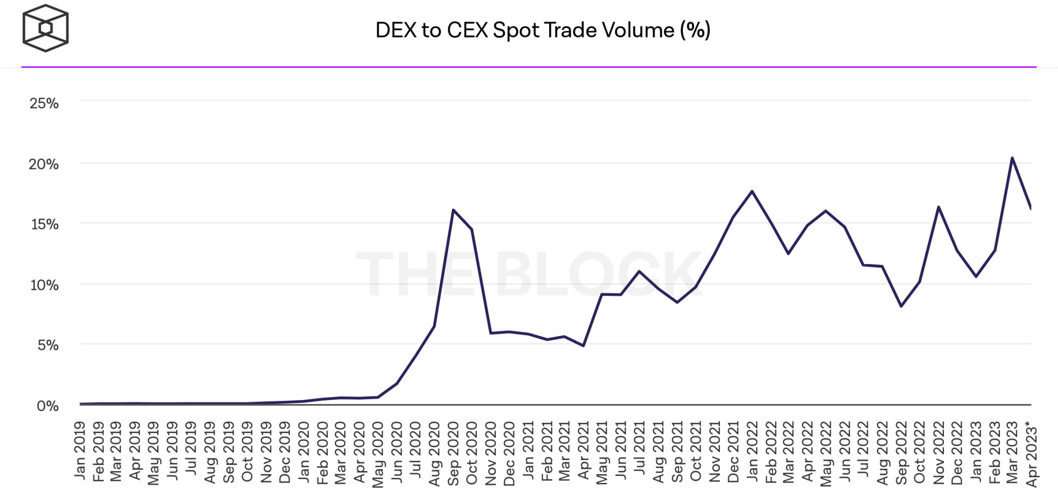
Centralized vs. decentralized exchanges: A quick compariosn

	Centralized exchange	**Decentralized exchange**
Custody of assets	Exchange	**User**
Security	Variable; risk of hacks	**High**
Trading mechanism	**Limit order book**	Automated market maker
Transaction speed	**Very fast**	Slow on-chain trades
Fees	Set by exchange	Set by exchange + network costs.

Decentralized exchanges (DEX) trade over US$100bn each month



Decentralized exchanges ≈ 20% of crypto trading volume



SOURCES: THE BLOCK, THE GRAPH, COINGECKO
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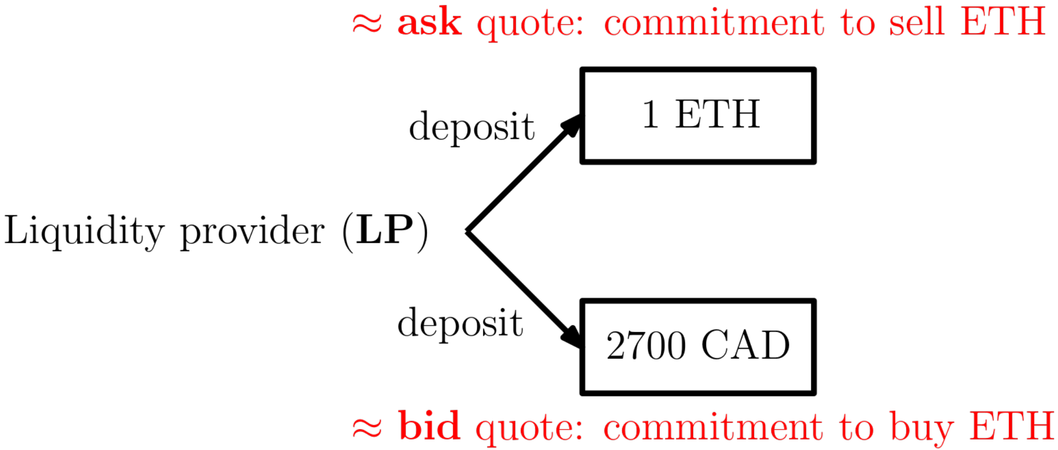
Liquidity provision on Uniswap v3: A simplified mechanism



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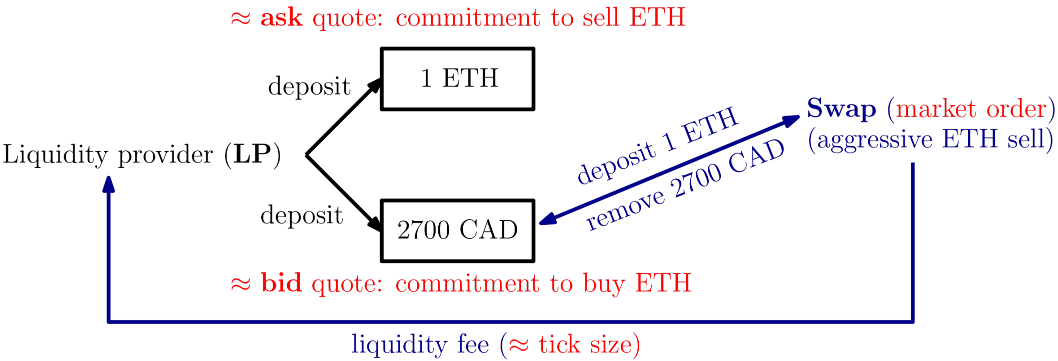
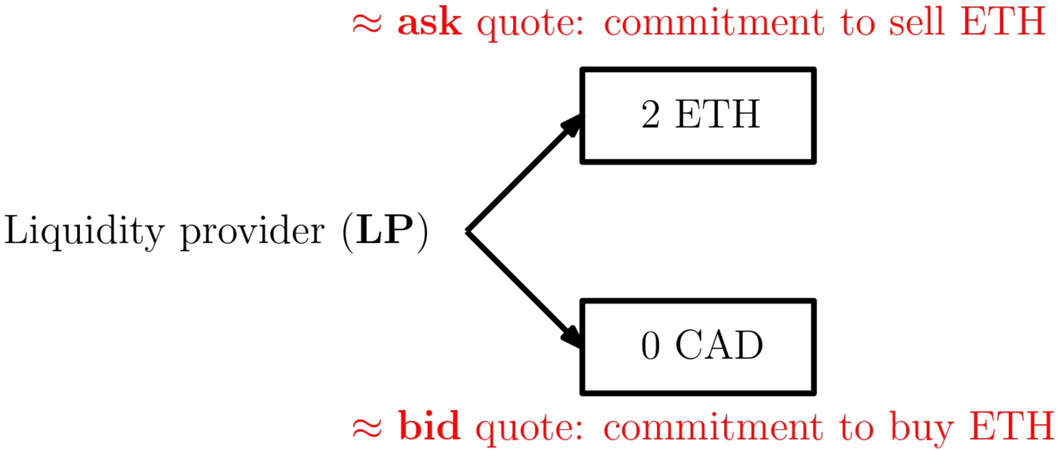


Liquidity provision on Uniswap v3: A simplified mechanism



Why is this interesting?

1. DEX mechanism are designed for **passive** liquidity provision.
   - Bid (ask) orders automatically convert to ask (bid) quotes after execution.
   - Liquidity is not removed, just transformed (in a potentially sub-optimal position).
2. Passive liquidity provision leads to lower:
   - 2.1 *gas costs* from interacting with Ethereum blockchain.
   - 2.2 *time/effort costs* of monitoring markets for smaller or retail liquidity providers.

Our paper finds that:

1. Passive and active LPs self-select in market with different fees. (*tick sizes*).
2. Passive LPs interact with a disproportionately small share of trading volume.

Related literature

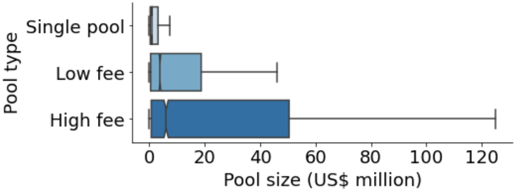
We contribute to:

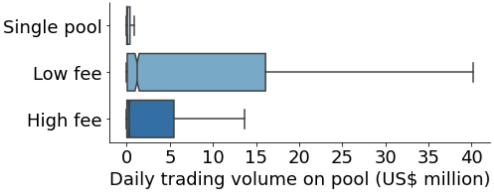
- a growing literature on decentralized exchanges (Lehar and Parlour, 2021; Park, 2022; Capponi and Jia, 2021; Aoyagi, 2020; Aoyagi and Ito, 2021; Barbon and Ranaldo, 2021; Hasbrouck, Rivera, and Saleh, 2022; Foley and Krekel, 2023).
- the literature on market fragmentation (Pagano, 1989; Pagnotta and Philippon, 2018; Foucault and Menkveld, 2008).
- the literature on the role of tick sizes on liquidity provision (Foucault, Kadan, and Kandel, 2005; Yao and Ye, 2018; Li, Wang, and Ye, 2021)

A puzzle: Liquid markets have lower volume

Each Uniswap pair can be traded on up to four markets ("liquidity pools"):
Liquidity fees can be either 1, 5, 30, or 100 bps.

1. **Stylized fact #1:** The largest pairs (ETH-stablecoins, ETH-BTC ...) are more likely to become fragmented.
2. **Stylized fact #2:** For a given pair, the low-fee pool attracts higher volume. (consistent with LP competition and optimal order routing)
3. **Stylized fact #3:** For a given pair, the high-fee pool attracts more liquidity.





Outline

Model

Empirical findings

Model

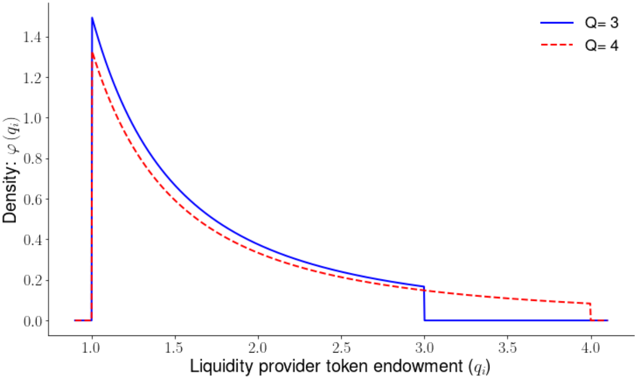
Asset and markets.

A single asset with expected value v trades on two liquidity pools with fees $h > \ell > 0$.

Liquidity providers (**LP**)

- Risk-neutral;
- Token endowments q_i;
- q_i follows a bounded Pareto distribution:

$$\varphi(q) = \frac{Q}{Q-1}\frac{1}{q^2}.$$



Model

Liquidity takers (**LT**).

Two types of **LT**:

1. *small* **LT** arrive at constant rate $\theta \, \mathrm{d}t$ and optimally go to the low-fee pool first (ℓ).
2. *large* **LT** demand Θ token units and arrive as Poisson process $J_t(\lambda)$.
   They are exogenously large enough to consume all liquidity on ℓ and h pools.

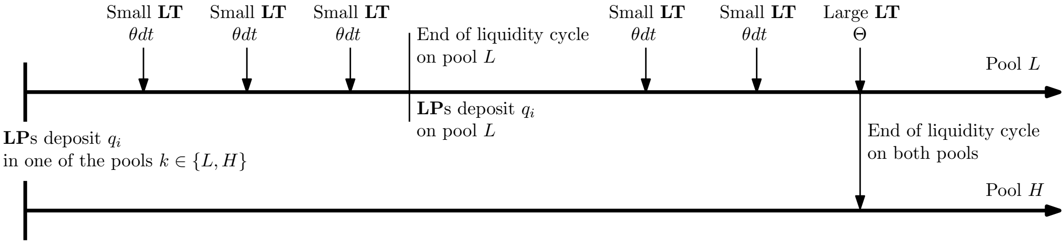
Liquidity demand:

$$\mathrm{dLiquidityDemand}_t = \theta \, \mathrm{d}t + \Theta \, \mathrm{d}J_t(\lambda) \, ,$$

Gas costs.

Any interaction with a liquidity pool has a fixed cost $\Gamma > 0$.

Model timing and liquidity cycles



Equilibrium

- Liquidity providers choose pool $k^\star$ to maximize expected profit per unit of time:

$$k^\star(q_i) = \arg\max_k (q_i f_k - \Gamma) \frac{1}{d_k}, \text{ where } f_k \in \{\ell, h\}$$

- d_k is the endogenous liquidity cycle duration, which $\nearrow$ in aggregate liquidity:

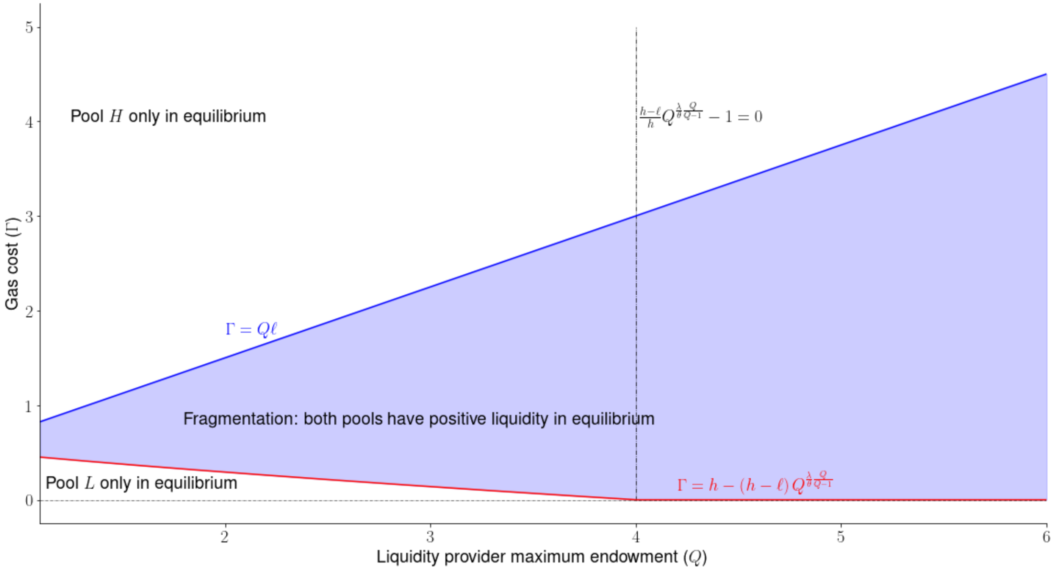
$$d_L = \frac{1}{\lambda} - \frac{1}{\lambda} \exp\left(-\frac{\mathcal{L}_L}{\theta}\lambda\right) \text{ and } d_H = \frac{1}{\lambda},$$

where $\mathcal{L}_L = \int_{i \in \Omega_L} q_i \varphi(q_i)$ is the aggregate liquidity on the low-fee pool.

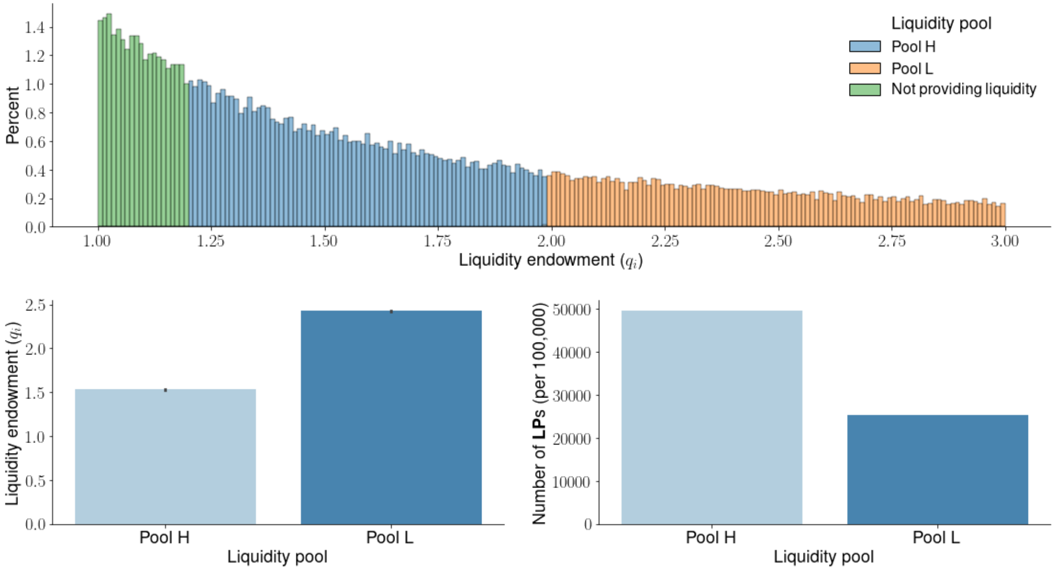
- The marginal **LP**'s endowment solves:

$$q^\star_{mg} = \frac{d_H - d_L}{d_H \ell - d_L h} \Gamma$$

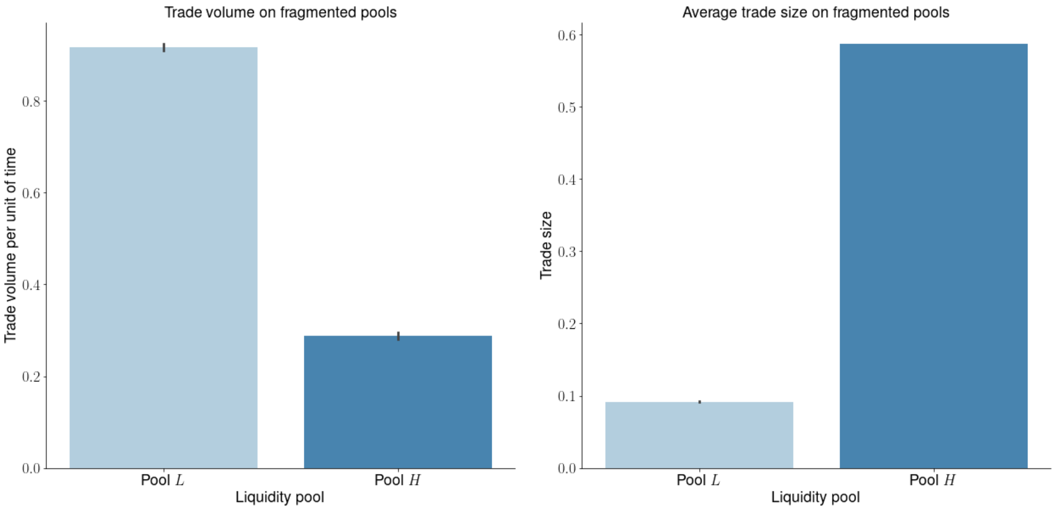
Equilibrium regions



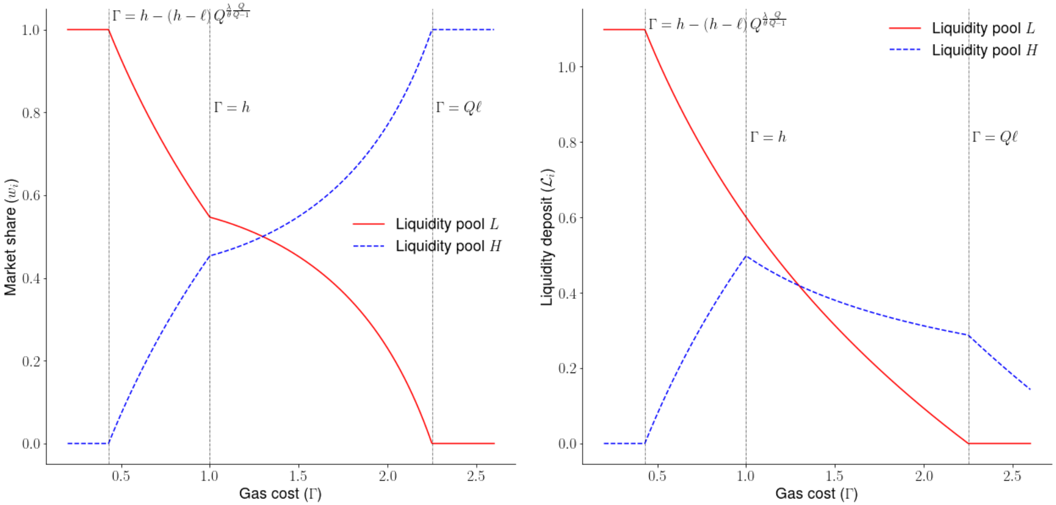
High-fee pools attract small liquidity providers



Trading volume and liquidity in equilibrium



Gas cost and liquidity market shares



Outline

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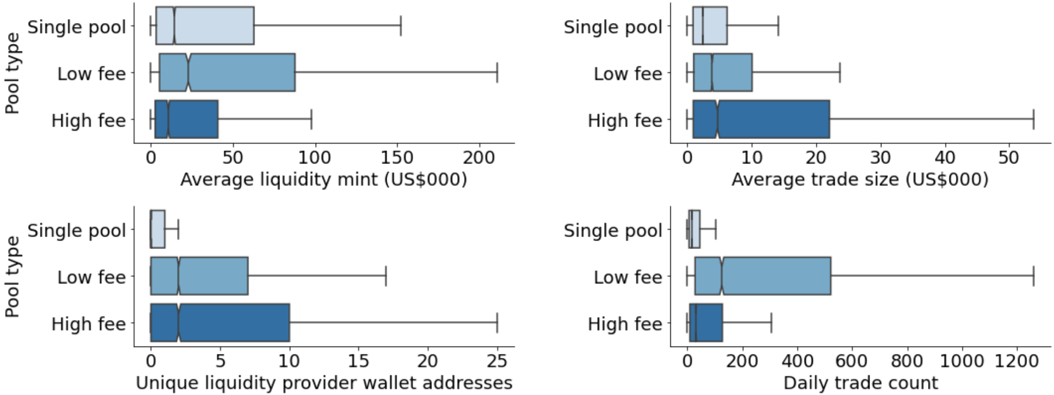
Data

- Data from Kaiko on all Uniswap v3 trades, liquidity deposits and withdrawals from May 4, 2021 until September 15, 2022, including traders' wallet addresses.
- Convert all token prices into USD using a minute-by-minute Kaiko Cross-Price API.
- Gas cost is the average of the lowest daily 100 gas prices for mint events.
- Focus on economically sizeable pools:
  1. active in more than 30 days within the sample;
  2. 100+ liquidity events throughout the sample;
  3. average daily liquidity balance $>$ US$100,000;
  4. $>$1% of volume for a traded pair.
- We obtain 262 pools in 224 asset pairs:
  1. aggregate daily volume of US$ 1.32bn;
  2. end-of-sample aggregate liquidity US$ 3.07bn.
  3. account for 87.56% of all Uniswap v3 interactions.

Liquidity clienteles: high fee pools feature many small **LP**s.



Fragmentation and order flow characteristics

	Mint size	Trade size	Volume	# Trades	# Wallets	LP interactions
$d_{\text{low-fee}}$	0.65***	-0.32***	1.08***	1.19***	-0.20***	-0.18***
	(9.76)	(-14.49)	(19.86)	(47.70)	(-7.71)	(-5.91)
Gas price $\times$ $d_{\text{low-fee}}$	0.38***	0.12***	-0.03	-0.15***	-0.21***	-0.24***
	(4.05)	(4.22)	(-0.80)	(-5.65)	(-10.06)	(-9.76)
Gas price $\times$ $d_{\text{high-fee}}$	0.54***	0.16***	0.21***	-0.01	-0.12***	-0.11***
	(5.86)	(6.73)	(4.06)	(-0.28)	(-4.19)	(-3.38)
Trade volume (pair)	0.69***	0.30***	0.73***	0.38***	0.06***	0.12***
	(8.12)	(14.84)	(15.87)	(12.88)	(3.44)	(5.09)
Pool size (pair)	-0.58*	-0.03	-0.07	-0.28***	-0.08	-0.15**
	(-1.97)	(-0.17)	(-0.34)	(-3.38)	(-1.32)	(-2.23)
Volatility	-0.00	-0.03***	0.05	0.11**	0.01	0.02
	(-0.03)	(-3.39)	(0.62)	(2.58)	(0.65)	(1.38)
Constant	-2.67	-2.00	-4.08***	0.54	1.14**	1.24***
	(-1.08)	(-1.50)	(-2.72)	(1.01)	(2.62)	(2.69)
Observations	11,695	20,454	20,454	20,454	20,454	20,454
R-squared	0.26	0.55	0.59	0.67	0.62	0.60

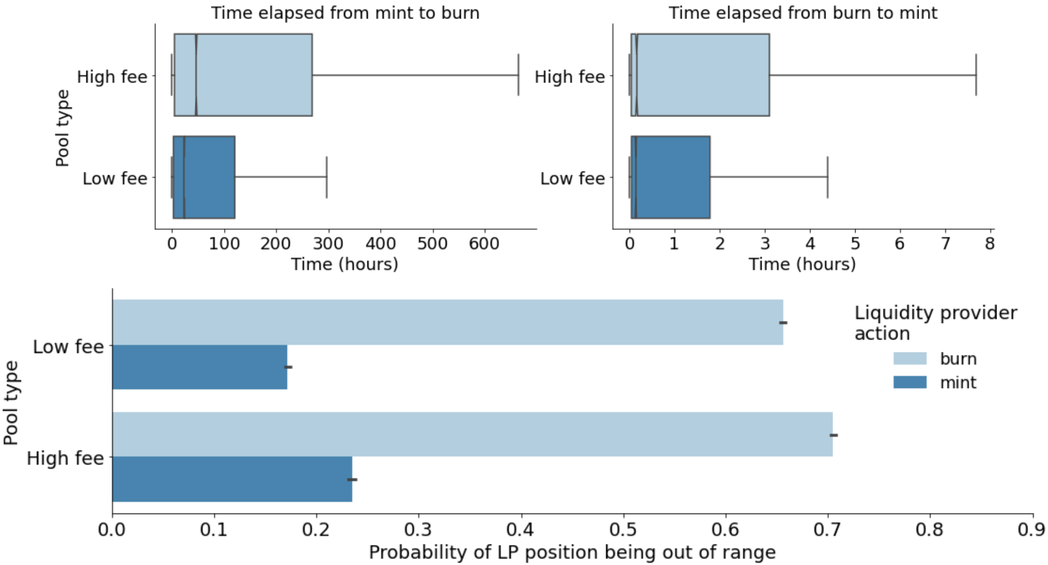
Do gas prices move market shares?

	Liquidity market share (%)			Volume market share (%)		
$d_{\text{low-fee}}$	-11.57***	-11.82***	-11.57***	29.01***	28.49***	29.01***
	(-16.82)	(-16.18)	(-16.82)	(25.95)	(23.43)	(25.92)
Gas price $\times$ $d_{\text{low-fee}}$	-2.30***	-2.02**	-2.30***	-2.30*	-1.37	-2.30*
	(-3.19)	(-2.64)	(-3.19)	(-1.77)	(-0.99)	(-1.77)
Gas price	1.26***	1.09***	1.30***	1.28*	0.80	1.50**
	(3.32)	(2.68)	(3.43)	(1.95)	(1.14)	(2.23)
Trade volume (pair)	-0.28**	-0.31**	-0.34**	-0.54***	-0.58***	-0.91*
	(-2.57)	(-2.53)	(-2.00)	(-3.61)	(-3.43)	(-1.72)
Pool size (pair)	-0.77*	-1.10**		-4.56***	-5.11***	
	(-1.74)	(-2.45)		(-3.54)	(-4.10)	
Volatility	0.06**		0.06**	0.02		0.02
	(2.54)		(2.53)	(0.87)		(0.78)
Constant	65.84***	68.95***	60.58***	79.44***	84.64***	48.53***
	(15.85)	(15.87)	(24.95)	(7.21)	(7.72)	(6.44)
Observations	20,454	21,097	20,454	20,454	21,097	20,454
R-squared	0.03	0.03	0.03	0.13	0.13	0.13

Liquidity flows and gas prices

	Daily mints (log US$)			Prob (at least one mint)		
$d_{\text{low-fee}}$	0.15*	0.16**	0.15*	1.33*	1.30*	1.33*
	(1.94)	(2.03)	(1.94)	(1.82)	(1.85)	(1.82)
Gas price × $d_{\text{low-fee}}$	-0.36***	-0.36***	-0.39***	-7.60***	-7.63***	-5.68***
	(-6.66)	(-6.43)	(-5.22)	(-9.36)	(-9.09)	(-8.22)
Gas price × $d_{\text{high-fee}}$	0.03	0.00		-1.92***	-2.14***	
	(0.33)	(0.00)		(-2.74)	(-2.85)	
Trade volume (pair)	0.45***	0.44***	0.45***	1.19	1.17	1.19
	(7.16)	(7.04)	(7.16)	(1.33)	(1.25)	(1.33)
Pool size (pair)	-0.45***	-0.52***	-0.45***	-5.31**	-5.56**	-5.31**
	(-2.75)	(-3.34)	(-2.75)	(-2.43)	(-2.52)	(-2.43)
Volatility	0.02		0.02	1.50*		1.50*
	(0.73)		(0.73)	(1.80)		(1.80)
Gas price			0.03			-1.92***
			(0.33)			(-2.74)
Constant	0.55	1.14	0.55	81.06***	82.73***	81.06***
	(0.60)	(1.36)	(0.60)	(6.12)	(5.72)	(6.12)
Observations	20,454	21,097	20,454	21,097	20,454	20,454
R-squared	0.51	0.51	0.51	0.61	0.62	0.62

Evidence of heterogeneous liquidity cycles



Liquidity cycles on low- and high-fee pools

	Mint-burn time			Burn-mint time		
$d_{\text{low-fee}}$	-77.26***	-95.74***	-99.63***	-117.18***	-132.22***	-132.65***
	(-8.53)	(-10.46)	(-11.01)	(-9.76)	(-10.49)	(-10.53)
Gas price $\times$ $d_{\text{low-fee}}$	-30.43***	-33.90***	-33.62***	-10.03	-13.01*	-12.93*
	(-3.76)	(-4.04)	(-4.02)	(-1.61)	(-1.88)	(-1.86)
Gas price $\times$ $d_{\text{high-fee}}$	-16.84***	-9.75*	-9.13	-1.08	0.45	0.53
	(-2.99)	(-1.77)	(-1.67)	(-0.20)	(0.07)	(0.08)
Trade volume (pair)		1.46	-1.04		-6.54	-7.01
		(0.18)	(-0.13)		(-0.76)	(-0.82)
Pool size (pair)		73.87	80.68		-74.14*	-73.89*
		(1.05)	(1.18)		(-1.72)	(-1.71)
Volatility		3.37	2.70		-50.19***	-50.23***
		(0.17)	(0.14)		(-5.57)	(-5.59)
Position out-of-range			46.80***			14.39**
			(8.60)			(2.27)
Constant	389.08***	-174.79	-222.32	150.80***	831.58**	833.67**
	(110.18)	(-0.30)	(-0.39)	(29.01)	(2.39)	(2.40)
Observations	287,505	265,182	265,182	196,145	182,581	182,581
R-squared	0.82	0.82	0.82	0.37	0.38	0.38

Conclusion

- Decentralized exchanges encourage *passive* liquidity provision, both to reduce gas costs and encourage smaller traders to participate as market makers.
- However, fixed costs to participate in markets lead to different economies of scale for heterogeneous **LP**s.
- Market-maker clienteles emerge if trading is fragmented across different-fee pools.

Low-fee pools	High-fee pools
High trading volume	Low trading volume
Low aggregate liquidity	High aggregate liquidity
Few, large **LP**s	Many, small **LP**s
Short liquidity cycles	Large liquidity cycles