Do high-frequency market makers share risks?

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Motivation

- Majority of stock exchanges world-wide are organized as limit order book markets
  - Rank incoming limit orders in a queue by **price-time priority**
- Market makers (**MM**) supply liquidity by submitting limit orders to the book
- The sequential nature of the book implies that **MM**s absorb incoming trades one by one, leading to **imperfect risk sharing**
  - As compared to a double auction for example
  - Contrasts the literature which typically assumes that the market making sector absorbs incoming trades and share risks

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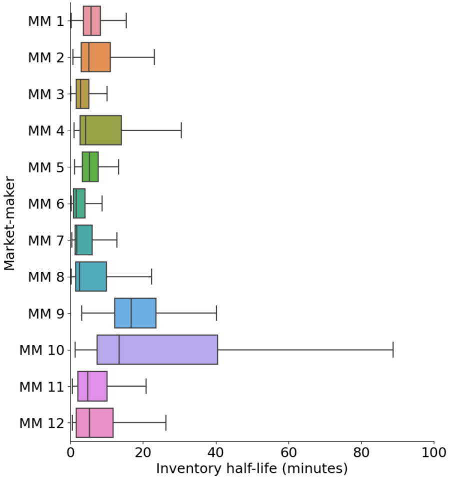
  - As compared to a double auction for example
  - Contrasts the literature which typically assumes that the market making sector absorbs incoming trades and share risks
- How well do market makers share risks? How do time priority and inventory position affect market maker limit order strategies? What is the impact on limit order sizes and quoted depth?

Overview talk

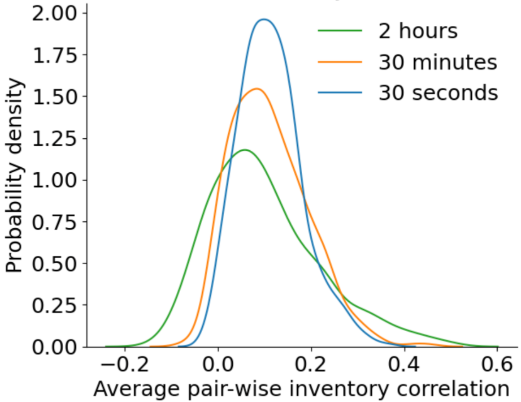
- We start with empirically documenting **three stylized facts** on patterns in market maker inventories
  - Using data from 2021:01-2021:08 of the TMX Montréal Exchange, identify **12 market makers** in the Bond and Equity futures market
- Offer a theoretical model to study **optimal limit order sizes**, in a market with price-time priority and imperfect competition
- Empirically, novel identification approach of the size of **inventory and adverse selection frictions**, as well as the cost of imperfect **MM** risk sharing

Stylized fact 1: Inventory risk matters



Further, **MM**s end the day with a flat position for 72% of instruments and days.

Stylized fact 2: do **MM**s share risks?



- Pairwise inventory correlation is low, for 66 **MM** pairs * # days * # contracts.

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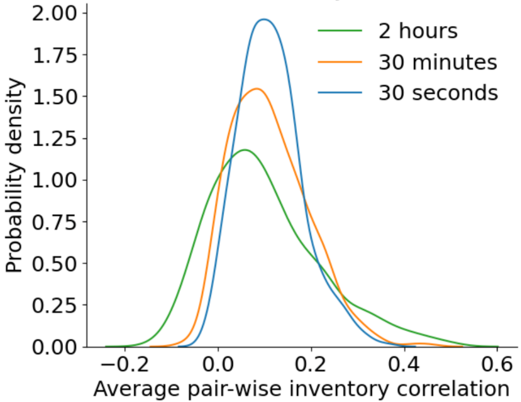

- Pairwise inventory correlation is low, for 66 **MM** pairs * # days * # contracts.
- Only 12,4% of volume is between HFTs; compared to 24% for LSE dealers in the 90s and 28-42% for corporate bond dealers Reiss and Werner (1998); O'Hara and Zhou (2021)

Stylized fact 3: Does queue position depend on inventory?

A long **MM** wants to be early in the queue on the ask side to unwind. Can he manage this?

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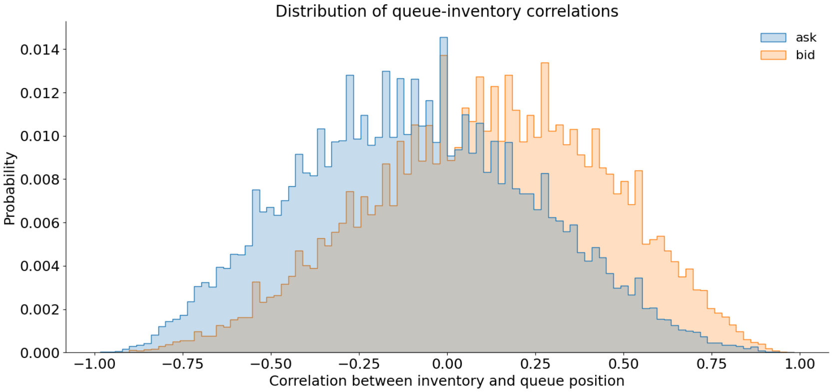

A striking picture

1. **MMs** care deeply about managing inventory positions (half-life $<$15 minutes; end 72% of days neutral), yet...
2. they have low inventory correlations, suggesting poor risk sharing, and...
3. in the queue of limit orders, the **MMs** most eager to trade seem unable to get priority

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2. they have low inventory correlations, suggesting poor risk sharing, and...
3. in the queue of limit orders, the **MMs** most eager to trade seem unable to get priority

How do queue position and inventory affect limit order sizes? Does the arrival sequence affect aggregate depth? Can we quantify this risk-sharing inefficiency?

Our contribution

Theory

A limit order book model with: i) adverse selection, ii) **MM** risk aversion, iii) discrete prices with time priority, iv) imperfect competition between **MM**s.

1. Time priority and sequential execution $\rightarrow$heterogeneous **MM** inventories and imperfect risk sharing!
2. A larger limit order early in the queue raises adverse selection of subsequent orders $\rightarrow$crowding-out effect. Queuing order matters.
3. Trade-off between depth and **MM** risk-sharing in time-priority markets.

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Empirical

1. A simple OLS regression estimates the theoretical model →structural interpretation identifies **adverse selection** and **inventory costs** through limit order sizes
2. Quantify the **risk sharing inefficiency**
3. Arrival sequence of MM impacts quoted depth.

Related literature

We contribute to various strands of literature

- Risk sharing among market makers (Reiss and Werner, 1998; Comerton-Forde, Hendershott, Jones, Moulton, and Seasholes, 2010)
- Limit order book models used to estimate adverse selection (Sandås, 2001; Hollifield, Miller, and Sandås, 2004)
- Identification of inventory frictions through price pressures (Kraus and Stoll, 1972; Brogaard, Hendershott, and Riordan, 2014; Hendershott and Menkveld, 2014)
- The role of the tick size on liquidity provision (Yao and Ye (2018), Li, Wang, and Ye (2021))
- Order book priority rules and optimal market design (Degryse and Karagiannis (2019), Budish, Cramton, and Shim (2015))

Model

Asset

- Single risky asset paying $\tilde{v}$ at $t = 2$, with $\mathbb{E}_0 \tilde{v} = v$.

Market

- One-tick market (Parlour, 1998) with two prices: $p_{-1} < v < p_1$ and time priority.

Market makers

- $J \geq 2$ market makers (**MM**) with starting inventory I_j, who maximize:

$$\mathbb{E}_0 \left[\text{Wealth}_{t=2} - \frac{\gamma}{2} I_{t=2}^2 \right].$$

Impatient traders

- Market orders x at $t = 1$ from exponential distribution with mean ϕ.
- Exogenous price impact as in Sandås (2001): $\mathbb{E}\left[\tilde{v} \mid x\right] = v + \lambda x$.

Model timing

t=0

1. **MM** $j = 1$ submits limit orders (Q_1^{bid}, Q_1^{ask}).
2. **MM** $j = 2$ submits limit orders (Q_2^{bid}, Q_2^{ask}).
3. **MM** $j = 3$ submits limit orders (Q_3^{bid}, Q_3^{ask}).

...

J. **MM** $j = J$ submits limit orders (Q_J^{bid}, Q_J^{ask}).

t=1

1. Impatient trader submits marketable order of size x.
2. The market clears.

t=2

Payoffs realized

Optimal liquidity provision

Equilibrium condition: The marginal limit order by market maker j earns zero expected profits:

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$$Q_j^\star = \underbrace{\frac{p_1 - v - \lambda\phi}{\gamma}}_{\substack{\text{Risk-adjusted}\\\text{pie}}} \left(\frac{\gamma}{\gamma+\lambda}\right)^j + \underbrace{\frac{\gamma}{\gamma+\lambda} l_j}_{\substack{\text{direct}\\\text{effect}}} - \underbrace{\sum_{k=2}^{j}\left(l_{j-k+1}\frac{\lambda}{\gamma}\left(\frac{\gamma}{\gamma+\lambda}\right)^{k-1}\right)}_{\text{crowding-out effect if } j\geq 2}.$$

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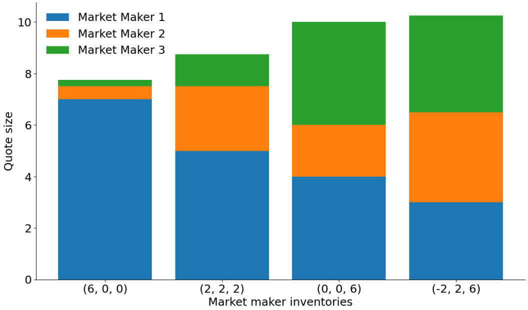
Aggregate depth $\overline{Q}_J$ depends on arrival sequence of **MM** inventories:

$$\overline{Q}_J = \sum_{k=1}^{J} \left[\left(\frac{p_1 - V - \lambda v \phi}{\gamma} + I_{J-k+1} \right) \left(\frac{\gamma}{\gamma+\lambda} \right)^k \right].$$

Sequence of arrival and aggregate depth



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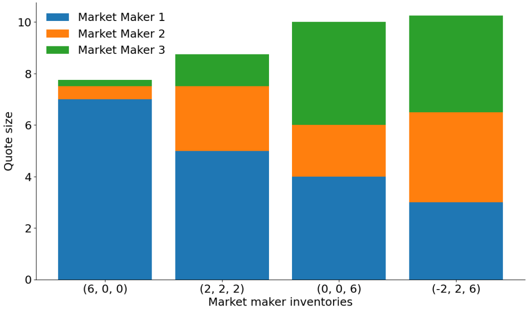

Empirically we ask:

1. Do individual limit order sizes depend on inventory and queue position?
2. Can we identify inventory and information frictions from limit order sizes?
3. Can we quantify the risk sharing inefficiency?
4. Does depth depend on queuing arrival sequence?

Discussion

Strong assumptions:

1. Static model with only a single bid and ask price level
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Strengths:

1. Retain important queuing dynamics
2. Imperfect competition between market makers
3. Interaction between adverse selection and inventory frictions
4. Simple and closed form solution that we can take to the data

Data

1. All trades and 30-seconds BBO snapshots from TMX Montréal Exchange for:
   - Ten- and Five-Year Government of Canada Bond Futures
   - S&P/TSX Equity Index Standard Futures
2. Includes trader and limit order submitter IDs.
3. Includes all lit and iceberg orders.
4. Includes queue position for each order at the BBO.
5. Sample covers January 1st to August 18, 2021.
6. The TMX Montréal Exchange has strict price-time priority.

We identify 12 market maker accounts

We follow the literature (Kirilenko, Kyle, Samadi, and Tuzun, 2017):

- Trade a lot: participate in $>$50 trades per day-instrument.
  vspace0.1in
- Do not built significant positions: The end-of-day net position $\leq 5\%$ of daily traded volume.
- Mean-revert positions often during the day: The average squared deviation from end-of-day position divided by dollar volume $\leq 2.5\%$
- Fourth (own criteria): Have a quote at the BBO in at least 20% of the snapshots.

Summary statistics at the contract-day-trader level

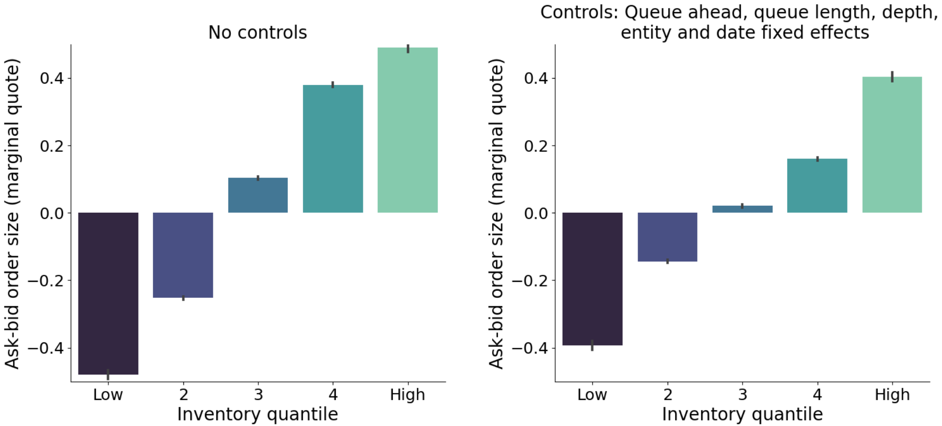
MMs	Trade count	Volume	Net pos. (%)	σ_{Inv} (%)	Time at BBO (%)
Mean	2,025.91	3,870.19	0.42	0.91	52.92
St. Dev.	2,321.61	4,833.90	2.66	2.32	19.53
Pctl(25)	323	621.5	0	0.16	33.17
Median	1,190	1,792	0	0.37	51.57
Pctl(75)	3,006.8	5,641.8	0.05	0.82	69.31

non-**MMs**	Trade count	Volume	Net pos. (%)	σ_{Inv} (%)	Time at BBO (%)
Mean	82.70	229.78	73.25	55.50	1.92
St. Dev.	258.77	802.75	38.61	31.00	4.32
Pctl(25)	3	5	39.3	27.0	0.14
Median	14	25	100	66.1	0.53
Pctl(75)	62	138	100	78.7	1.92

Question 1:

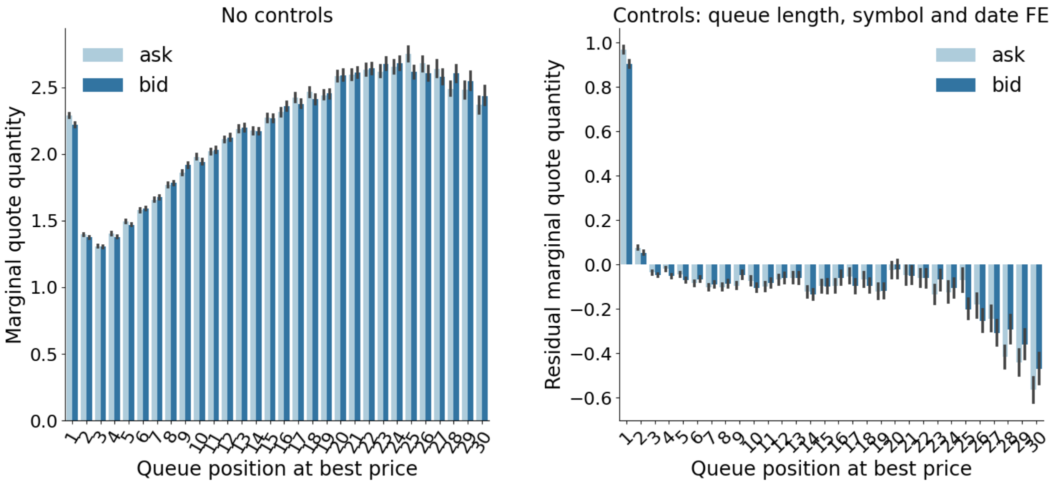
Do individual limit order sizes depend on i) inventory and ii) queue position?

Does inventory matter for quote sizes?



Longer inventory position means a larger Ask and smaller Bid quote.
Note: average limit order size is 1.84 contracts!

Does queue position matter for quote sizes?



Question 2:

Can we identify inventory and information frictions from limit order sizes?

Inventory concerns and adverse selection in quote sizes

The model allows for a structural estimation by OLS, identifying $-\frac{\lambda}{\lambda+\gamma}$ and $\frac{\gamma}{\lambda+\gamma}$.

The near-zero correlation between queue position and inventory offers clean identification

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The near-zero correlation between queue position and inventory offers clean identification

	Quote size				
	(1)	(2)	(3)	(4)	(5)
queue ahead $\times d_{\text{side}}$	-0.146^{***}	-0.146^{***}	-0.145^{***}	-0.144^{***}	
	(-26.431)	(-26.443)	(-26.127)	(-25.330)	
order priority $\times d_{\text{side}}$					-0.034^{***}
					(-14.463)
Inventory	0.152^{***}	0.152^{***}	0.153^{***}	0.153^{***}	0.144^{***}
	(34.501)	(34.502)	(34.540)	(34.526)	(33.250)
queue length	0.071^{***}	0.071^{***}	0.071^{***}	0.071^{***}	0.086^{***}
	(53.066)	(53.068)	(54.999)	(57.312)	(39.906)
book depth $\times d_{\text{side}}$	0.009	0.009			-0.016^{*}
	(1.024)	(1.022)			(-1.859)
order imbalance $\times d_{\text{side}}$	0.018				
	(1.604)				
Adjusted R^2	0.234	0.234	0.234	0.234	0.235

Question 3:

Can we quantify the risk sharing inefficiency?

Risk sharing inefficiency: back-of-the-envelope calculation

Divide *actual* squared holdings summed over all **MM**s by the sum under perfect risk sharing.

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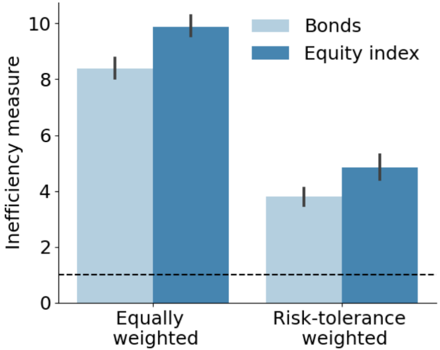
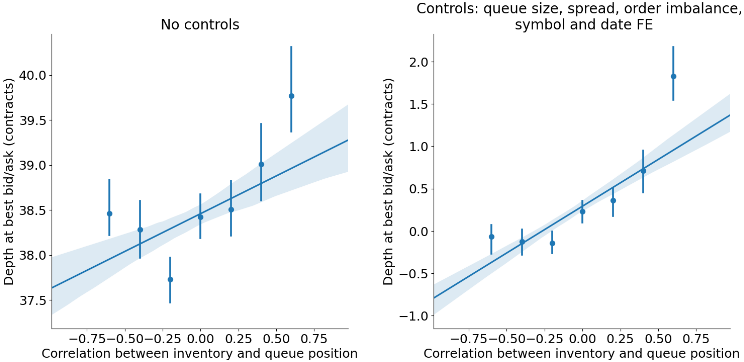


If the inventory penalty is inversely proportional to the time-series SD of holdings, the market wide inventory cost could be 4 times lower under perfect risk sharing.

Question 4:

Does quoted depth depend on the queuing arrival sequence?

Sequence of arrival matters for quoted depth



Tradeoff: A high correlation between inventory and queue position is bad for **MM**s, since the one most eager to trade is last in the queue, yet increases quoted depth

Quoted depth and arrival sequence

	Market-maker quoted depth (contracts) on book side				
	(1)	(2)	(3)	(4)	(5)
$\hat{\rho}$(queue, inventory) $\times d_{\text{side}}$	0.520	0.981**	0.972**	1.100***	
	(0.701)	(3.617)	(3.543)	(4.356)	
$\hat{\rho}$(queue, inventory) $\times d_{\text{ask}}$					1.118***
					(4.207)
$\hat{\rho}$(queue, inventory) $\times d_{\text{bid}}$					−1.082**
					(−3.960)
queue size		1.772***	1.772***	1.693***	1.693***
		(16.927)	(16.906)	(18.348)	(17.955)
quoted spread (bps)				11.534***	11.534***
				(8.738)	(8.738)
	Symbol, date, trader FE				
Adjusted R^2	0.188	0.677	0.678	0.690	0.690

Conclusions

- Crowding-out effect: a large limit order to unwind inventory increases adverse selection risk of subsequent MMs in the queue
- Queuing sequence generates a trade-off between risk sharing and liquidity provision
- The risk sharing inefficiency is a welfare loss (holding fixed the ambiguous impact on quoted depth which depends on the queuing sequence)

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- Queuing sequence generates a trade-off between risk sharing and liquidity provision
- The risk sharing inefficiency is a welfare loss (holding fixed the ambiguous impact on quoted depth which depends on the queuing sequence)
- Empirically, if queue position $\uparrow$ 1 s.d. then quote size $\downarrow$ 0.146 contracts (7.93%)
- Empirically, if inventory $\uparrow$ 1 s.d. then quote size $\downarrow$ 0.152 contracts (8.2%)
- Arrival sequence randomness generates up to 8.4% variation in market depth due to crowding out.
- Inventory positions diverge significantly; costs could be four times lower under perfect risk sharing.