Tiny Trades, Big Questions: Fractional Shares†

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Abstract

This paper investigates fractional share trading. We develop a methodology for identifying fractional share trades in the Consolidated Transaction Reporting System. Our approach uses a latency-based digital footprint to estimate fractional share trades executed by Robinhood and Drivewealth, the two largest fractional share broker dealers. We find a surprising breadth to fractional share trading: high-priced stocks, meme stocks, IPOs, SPACs, and other popular retail stocks now exhibit considerable numbers of these tiny trades. We show that these tiny trades matter: fractional share trades are predictive of future liquidity and volatility, suggesting an information content to fractional share trades. Our results indicate that our measure of fractional share trading better captures this market information than do standard measures of retail trading. We also discuss how current data and reporting protocols preclude knowing the full extent of fractional share trading, inflate trading volume data, and provide at best censored samples of these off-exchange trades.

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1. Introduction

Retail trading in equities markets is enjoying a renaissance. No longer the "quirky side show" of years past, estimates of retail trading rose from 20% of the market in 2010, to perhaps as high as 40% in 2021.[1] U.S. retail brokerage accounts increased from 59 million in 2019 to 95 million in 2021. Fidelity alone had 32.5 million accounts in 2021, with Charles Schwab reaching 29.6 million accounts and relatively new entrant Robinhood Markets hosting 13 million users. Causes of this retail resurgence are varied, but the entrance of fintech trading apps such as Robinhood and SoFi, the introduction of commission-free trading in 2018, and even the distribution of stimulus checks are noted as prime factors. So, too, is the introduction of fractional share trading in 2019 which allowed investors to purchase as little as $1 of high-priced stocks like Berkshire-Hathaway A and Tesla. This paper investigates fractional share trading with a particular focus on understanding both the scale and impact of this new innovation.

Fractional shares are not really new—and they were usually viewed as a problem. Traditionally, fractional shares arose as part of stock dividends that were paid out in shares (or scrips if arising from a stock split), with dividend reinvestment programs (DRIPs) the modern formulation of this practice.[2] An article in the New York Times in 1930 decried fractional shares as a "nuisance" which "clutter up" accounts, are expensive to service, and difficult to sell.[3] The modern incarnation of fractional share trading is very different in both motivation and practice. In 2019, brokerage firms Interactive Brokers and Robinhood set up dedicated fractional share trading operations to allow retail customers to invest a specific amount of money in a stock rather than buy a specific number of shares. Other major retail brokers quickly followed suit, paving the way for retail access to even the most expensive shares. This dollar-based purchasing

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[1] As we discuss later in the paper, determining the exact size of retail trading is difficult. Jeffries estimated U.S. retail trade in 2021 as 32%, Reuters put it at 30%, and the Economist at 40%. See "Just how mighty are active retail traders", Economist, Aug. 21, 2021 and "Factbox: The U.S. retail trading frenzy in number," Reuters, Jan. 29, 2021.

[2] See "IBM will use scrip in fractional shares," New York Times, January 29, 1946.

[3] The article notes that "of all the tedious and thankless tasks brokers perform for their customers, the one which causes them the most grief is disposing of fractional shares, or for that matter, the mere routine of carrying them on their books. It takes as much time, for instance, to keep books on 11/1500 of a share as on 1,000 shares" and selling them is "much more difficult…. Most houses charge a nominal commission which does not pretend to cover the trouble involved". See "Along the Highways of Finance Fractional Shares a Problem", The New York Times, April 27, 1930.

also set the stage for individual indexing, whereby retail traders can create personalized indexes largely composed of fractional shares.[4]

But like in times past, problems remain and even gauging the scale and scope of this fractional development is challenging. Fractional share trades essentially fall outside of the National Market System (NMS). No exchange will accept an order for a fractional share so all trading takes place in off-exchange venues. Fractional share trades are not included in the Rule 605 execution quality reports required of market venues so metrics such as transaction costs are not always easily determined. It is not even clear exactly how many fractional share trades occur due to the disparate clearing and reporting protocols that attach to these tiny transactions. As we discuss in the next section, there are two different methods of clearing fractional share trades, only one of which results in direct reporting to a trade reporting facility. However, the consolidated tape does not accept trade reports for less than a single share so even the fractional share trades that do report "round up" to one share even if the actual trade is for a vastly lower quantity. Bartlett, McCrary and O'Hara (2022) show how this FINRA (Financial Industry Regulatory Authority) rule resulted in volume on the tape being drastically over-stated for the Class A common stock of Berkshire Hathaway (BRK.A), causing the relationship between BRK.A and its paired stock BRK.B to break down, and introducing a variety of other negative effects on the market.

The first challenge then is how to figure out, at least for the direct reports to the consolidated tape, which trades are fractional and what determines the incidence of fractional share trading across stocks. Retail trades in general are not identified on the consolidated tape so one might expect that approaches to identifying retail trades could be applied to the fractional share trade problem. We show that this is not the case for fractional share trades, and we discuss why features of the market reduce the efficacy of these identification approaches in general. We develop a new methodology for fractional share trade identification using the "digital footprints" of one-share trades reported by the two largest fractional share brokers, Robinhood and Drivewealth (henceforth denoted RHDW trades). We use our approach and intra-day data to predict which trades are fractional, and we test our predictive model using weekly data from the FINRA OTC Transparency platform, which (as we show) can be used to track a portion of the

[4] For a discussion of how such direct indexing works see https://www.fidelity.com/learning-center/trading-investing/direct-indexing or https://www.schwab.com/direct-indexing.

fractional share trades that occur each week in the market. We show that our methodology performs well, giving researchers a new tool for detecting these tiny trades.

Our success in doing so allows us to address a variety of "big" questions. First, how important are fractional share trades and what determines their daily incidence across stocks? Are they really only used to access high-priced stocks or are fractional share trades ubiquitous across the market? How important are reported "rounded-up" one-share fractional share trades relative to one-share trades in general? Second, do these tiny trades actually matter in any meaningful sense for the market? Are they predictive of value relevant data such as future spreads and volatility?

Third, how well do the FINRA weekly OTC Transparency data capture fractional share trading? Are there biases in how it is calculated and if so how much do they matter? Does the over-counting of volume in the consolidated tape arising from fractional shares constitute a general problem for the market or is this problem largely confined to a subset of high-priced stocks?

Two overall conclusions emerge from our research. First, fractional share trades may be for small amounts but they can have a big impact on the market, both directly through what is fast becoming an important avenue for retail trade, and indirectly through their predictive relationship with future price movements. Second, and despite the new tools we develop here, there remains a remarkable lack of clarity regarding this growing market. Due to disparate reporting protocols, there is no way to know the total size of the market. Moreover, even the trade data that is available is flawed—the FINRA data is inaccurate due to censoring and the trades reported to the consolidated tape are inflated values. In the conclusion, we discuss the implications of our research for potential regulatory changes to improve transparency in this evolving market.

Our research is related to several strands of the literature. There is to date only a small literature looking at fractional share trades. Da, Fang, and Lin (2022) provide an interesting event study of the market impacts when four retail brokerage firms first introduce the ability to trade fractional shares. They find that fractional share trading generates price pressures and reversals for high priced stocks during attention generating events. Bartlett, McCrary and O'Hara (2022), in a companion paper, show how FINRA reporting rules resulted in severely inflated trading volumes for BRK.A, and affected liquidity and pricing in the market. They argue for changes to reporting rules and to the National Market System more generally to accommodate fractional share trading. Our work here demonstrates that fractional shares are not just used to buy tiny pieces of high-priced stocks—of the top 80 firms with the largest number of fractional share trades, nearly half have closing prices less than $20.00 per share. Moreover, while fractional share trades are individually tiny, we show that they are now collectively sizeable: For Tesla, the stock with the most fractional share trades identified by our methodology, 6.7% of all trades and 27% of all single-share trades reflect fractional share executions. Across all stocks in our sample, 1.5% of trades are fractional, with 13.2% of all single-share trades being fractional.

There is a much larger literature on retail trading. Particularly relevant for our paper is recent research on Robinhood trading and its impacts. Welch (2021) finds that Robinhood traders generally tilt towards high volume, high priced stocks with results suggesting both good timing and good alpha. Barber, Huang, Odean and Schwarz (2021) find that Robinhood traders engage in more attention-induced trading than other retail traders. Moss, Naughton, and Wang (2020) find that ESG disclosures are irrelevant to Robinhood traders' portfolio allocation decisions. Our analysis here provides the first look at the extensive use of fractional shares by Robinhood traders, as well as provides evidence on fractional share retail trading more generally.

A second strand of the literature relates to retail trade identification. Early attempts (see for example Lee and Radhakrishna (2000)) used small trade size as a proxy for retail orders. The rise of algorithmic trading and other market trends, however, renders such an approach untenable (see O'Hara (2015) or O'Hara and Ye (2014) for discussion). Boehmer, Jones, Zhang, and Zin (2022) (BJZZ) propose a metric that uses price improvement measured in small fractions of cents per share to identify retail trades. Using data from 2010, they show that this BJZZ metric accurately identifies retail trades arising from marketable orders. A recent paper by Barber, Huang, Jorion, Odean and Schwartz (2022) uses more recent data to argue otherwise. These authors find much lower accuracy in retail order identification using the BJZZ metric, and they propose modifications to the BJZZ approach. Neither approach, however, can identify retail fractional share trades and, as we discuss, the exclusion of mid-point trades and trades at the quotes by both approaches hampers their ability to do so.

Finally, there is a growing literature investigating whether retail trade is predictive of future price movements (see, for example, Barber, Odean, and Zhu (2009); Kaniel, Saar, and Titman (2008)). Jones, Zhang and Zhang (2022), using the BJZZ metric to identify retail trading in the pandemic, conclude that such trades are predictive of future stock prices. The latency-based digital footprint methodology we develop here provides an alternative metric to identify a subset of retail trade. We run a horse race between the predictive ability of the BJZZ metric-identified retail trades and the fractional retail trades identified by our metric for next day effective spreads, intraday volatility, and implied volatility. We find that fractional shares have much stronger predictability than the BJZZ metric, suggesting an even greater importance to the trades of retail traders (and underscoring the information content in fractional share trades). We believe the identification approach developed here can have broad applicability in investigating issues in retail trading.

This paper is organized as follows. The next section examines the challenges of finding fractional share trades in the data, setting out the current reporting rules and why many of these trades are captured in FINRA's weekly OTC Transparency data. We propose and test a new methodology using digital footprints to estimate fractional share trades in the NYSE Trade and Quote (TAQ) data. Section 3 uses this methodology to analyze fractional share trades in the cross-section of U.S. publicly-traded common stocks. Controlling for determinants of retail trade, we ask what types of stocks have greater cross-sectional retail fractional share trades. Section 4 then investigates the information content of fractional share trades, testing empirically whether RHDW fractional share trading is predictive of a stock's future liquidity and volatility. Section 5 addresses weaknesses with respect to fractional share trade reporting in current market data, investigating censuring in the FINRA OTC Transparency data arising from FINRA's *de minimus* rule as well as volume inflation in the consolidated tape arising from the rounding-up of fractional share trades. Section 6 summarizes our results and offers suggestions for changes needed to accommodate this new world of tiny trading. The Appendix provides more information regarding the classification rule we develop to identify fractional share trades in the TAQ data.

2. How Do You Find Fractional Share Trades?

Fractional share trading occurs when a customer places a trade that results in a trade confirmation indicating that the customer has acquired or sold a fraction of a whole share. In terms of the information available regarding fractional share trading, U.S. trade reporting rules create an uneven landscape at best. Some fractional share trades get reported to the public, but others do not, and those that do get reported are reported incompletely. There are a variety of reasons for this opacity, including alternative ways to report (or not report) trades, rounding rules that distort trade size, censored data due to activity-based disclosure standards, and non-compliance with FINRA rules. Despite these difficulties, we show how to develop classification rules for identifying in the TAQ data the fractional share trades for two of the most active retail brokers by using both a trade's digital "footprint" as well as publicly available FINRA data.

A. Why Studying Fractional Share Trades Is So Difficult.

An immediate problem for identifying fractional share trades is that retail broker-dealers take two different approaches to executing these trades for their customers, only one of which triggers clear public reporting obligations for the fractional share trade. The first approach, exemplified by Apex Clearing, to the best of our knowledge does not result in the fractional share trade being directly reported to the public tape. The second approach, exemplified by Robinhood and Drivewealth, does result in public reporting. We do not here weigh in regarding whether there is an economic or legal distinction between these two approaches. However, the approach taken clearly matters for any study of fractional share trading. Apex alone processes hundreds of millions of fractional share trades annually.[5] We next describe these two approaches in somewhat more detail.

The first approach to fractional share trading treats the fractional share trade as merely an accounting entry on the books and records of a brokerage firm, and thus not a "trade" or "transaction." For instance, many brokerage firms such as SoFi, Firsttrade, Betterment, M1 and Stash, among others, clear trades using Apex, which has facilitated fractional share trading since late 2018.[6] Under this approach, if SoFi seeks to execute a fractional buy for 3.2 shares of an issuer on behalf of a customer, Apex will execute this transaction by "rounding up" and purchasing four shares of the stock in the market. Apex then allocates 3.2 shares to SoFi's account and 0.8 shares to Apex's "fractional inventory account" that it manages through its

[5] Details concerning Apex's method for executing fractional share trades are provided in the registration statement relating to its (aborted) 2021 merger with Northern Star Investment Corp. II. See Northern Star Investment Corp. II, Amendment No. 2 to Form S-4 (filed May 24, 2021). As noted there, Apex disclosed that it currently offers fractional share investing in approximately 4,500 eligible equity securities and ETFs. It additionally disclosed that in 2018, 2019 and 2020, Apex processed 78,214,708, 129,821,441 and 208,637,634 trades that included a fractional share, respectively.

[6] See https://www.businesswire.com/news/home/20181023005447/en/, last accessed April 28, 2022.

proprietary trading desk.[7] As a result, the trade appears in the consolidated tape (and therefore the TAQ data) as a trade for four shares, but to the best of our knowledge no subsequent transactional report is made for the fractional share trade allocation. Nor is there a means to distinguish between whole share trades in the consolidated tape that include a fractional share trade component from those that do not.[8]

The second approach to fractional share trading involves a broker executing a fractional share trade on a principal basis, much like a retail market maker might internalize a whole share trade. This approach to fractional share trading is utilized by Robinhood, Fidelity, Charles Schwab, Interactive Brokers, and likely others, including the back-end brokerage firm Drivewealth.[9] Through its customizable suite of APIs, Drivewealth is the brokerage firm behind "micro-investing" platforms such Revolut and Cash-App that allow their customers to trade in fractional shares and/or invest their "spare change" in fractional shares.[10]

Critically, because a firm using the second approach executes a fractional share trade against its own inventory, each fractional share trade must be reported separately to a FINRA trade reporting facility, just as each trade internalized by a retail market maker must be reported

[7] See Northern Star Investment Corp. II, Amendment No. 2 to Form S-4 (filed May 24, 2021). As noted there, "Apex Fintech does not execute fractional share orders against its own inventory. When executing customer orders for fractional shares, after validating the order, Apex Fintech rounds the orders to the next whole share and sends a market (or limit, depending on the customer order received) order to the market. When the order is filled, the shares received are placed into Apex Fintech's fractional inventory account, whereby Apex Fintech then allocates the fractional shares to the customer's account and moves the residual, or otherwise unallocated fractional share, to Apex Fintech's own inventory account. Apex Fintech's inventory account is managed by the Apex Fintech trading desk. Typically, when the whole share quantities exceed internal quantity or notional thresholds, Apex Fintech reduces its positions to ensure Apex Fintech does not carry excessive risk."

[8] Fractional share trades executed in this fashion also do not appear separately in the OTC Transparency data. Whether this non-reporting approach to fractional share trading is congruent with FINRA's trade reporting requirements seems to be a question for FINRA. But it clearly obscures the total amount of fractional share trading in the market. On the other hand, we have argued elsewhere (Bartlett, McCrary, O'Hara 2022) that FINRA's "rounding-up" rule—which results in each fractional share trade being reported as a whole share trade on the consolidated tape and on the OTC Transparency platform—distorts the volume reported to the tape. We discuss in Section 6 potential reforms that might minimize these distortions.

[9] See, e.g., Fidelity, https://www.fidelity.com/trading/fractional-shares (When processing fractional and dollar-based orders, Fidelity Brokerage Services (FBS) will act as agent and National Financial Services (NFS) will act in a mixed capacity (as principal for the fractional share components and as agent for the whole share components) when executing an order."); Drivewealth, https://legal.drivewealth.com/fractional-shares-disclosure ("When executing on a Principal Basis, DriveWealth will execute the fractional component of the order against its principal facilitation account").

[10] For instance, Revolut will "round up" credit card transactions to the nearest whole increment, placing the difference between the rounded-up number and the actual credit card charge into a savings account that can be invested in fractional shares. Formally, platforms such as Revolut and Cash-App serve as "introducing brokers" and DriveWealth serves as the broker that executes and clears trades.

to a FINRA trade reporting facility.[11] Moreover, these brokers' customer agreements additionally provide that a trade involving both whole shares and fractional share trades (e.g., a trade for 3.2 shares, as in the above example) will be split into a whole share component and a fractional component, with the whole share component being executed on an agency basis. For this reason, the executing broker will typically be required to report a separate transaction record for the fractional component of the trade, as well as a transaction report for the whole share component.

Yet, while each fractional share trade executed using the second approach should be reported to the consolidated tape, actually identifying these fractional share trades faces a new challenge: the "rounding-up rule." Specifically, as documented in Bartlett, McCrary and O'Hara (2022), FINRA requires all fractional share trades to be "rounded up" to the nearest whole share, even if the fraction in question is genuinely tiny, such as one-millionth. This appears to reflect a preference for the status quo as the current consolidated reporting system only has the capacity to accept transaction reports for whole shares. As a result, a researcher looking solely at the consolidated tape for evidence of fractional share trades faces an immediate problem: How can she distinguish between those single share trades that represent fractional share trade executions from those that represent whole share executions?

In contrast to the first approach to fractional share trading, however, this second approach subjects these fractional share trades to additional public disclosures arising from FINRA's OTC Transparency initiative. In this paper, we exploit these additional disclosures to build two classification rules for identifying trade reports within the NYSE TAQ data that are likely to reflect fractional share trades executed by the two most active brokers using the second approach to fractional share trading—Robinhood and Drivewealth.[12]

B. The OTC Transparency Initiative

In 2016, FINRA launched a program to provide enhanced, publicly-available information about non-exchange trading activity. In particular, because all non-exchange trades are reported

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[11] The SEC has required since March 2007 that all off-exchange transactions be reported to a formal FINRA-managed Trade Reporting Facility. As described by O'Hara and Ye (2011), this requirement means that off-exchange trades made through a broker-dealer internalizer or in a dark pool (both of which were historically reported to an exchange and then consolidated with the exchanges' own trades when reported) are now effectively segregated and reported as having been executed at a FINRA TRF.

[12] As described below, the data fields submitted to the OTC (non-ATS) Transparency data include total trades and total shares (after application of FINRA's "rounding up" rule). This allows us to isolate those entities focused on fractional share trading, as the ratio of the two is for many entities almost exactly one. Inarguably, such entities are focused on fractional share trading. Of those entities, Robinhood and Drivewealth constitute about 80 percent of the reported volume.

to the consolidated tape with the same FINRA "D" venue code, the new OTC Transparency initiative was designed to provide greater insight into trading volume that occurs off exchanges in either an Alternative Trading Systems (ATS) or by a retail market maker. Under this program, FINRA uses the transaction reports that it receives from broker-dealers to assemble and publish on its website weekly summaries of non-exchange trading in U.S. equities. Among other things, these weekly summaries include the total number of trades and shares executed by each FINRA member on a stock-by-stock basis for trades executed on an ATS and for trades that are internalized by a market maker.[13] Because of FINRA's "rounding up" rule, these trades include fractional share executions internalized by a retail brokerage firm.

There are, however, two potential limitations to using these data to estimate the incidence of fractional share trading among brokers adopting the second approach to fractional share trading. (Brokers that adopt the first approach to fractional share trading do not appear at all in the OTC Transparency data for their fractional share trades.) The first relates to the "de minimis" trading threshold that applies to the disclosures.[14] Specifically, weekly trades will not be attributed to a specific FINRA member and will instead be aggregated and reported as completed by "De Minimis Firms" if either (a) the trades were executed by a FINRA member who executed fewer than 200 transactions per day for the reporting week, or (b) the trades were completed by a FINRA member who executed on average fewer than 200 transactions per day in the particular security during the week. In other words, a FINRA member must execute 1,000 or more transactions in a security over a 5-day trading week for the firm to be disclosed as trading that security in the OTC Transparency data.

The second limitation is one of compliance. In particular, the OTC Transparency data are based on the real-time trade reports submitted to a FINRA trade reporting facility; therefore, a firm's failure to report a trade to a TRF potentially undermines the reliability of these data. This issue has special significance in the context of fractional share trading because neither Robinhood nor Drivewealth appear to have understood FINRA's reporting rules for fractional share executions until well after they began their fractional share trading programs. Indeed, as discussed in Bartlett, McCrary, and O'Hara (2022), Robinhood expressly acknowledged this

[13] See FINRA Regulatory Notice 15-48, Equity Trading Initiatives: OTC Equity Trading Volume, available at https://www.finra.org/sites/default/files/Regulatory-Notice-15-48.pdf

[14] OTC Transparency (ATS and Non-ATS), Data Website User Guide January 16, 2019 Version 4, available at https://www.finra.org/sites/default/files/OTC-transparency-website-user-guide-v5.pdf

reporting deficiency—as well as its efforts to rectify it, upon being notified by FINRA of its obligations—in its 2021 annual report on Form 10-K. While we are not aware of a similar statement for Drivewealth, we provide evidence below indicating that Drivewealth also did not make real time trade reports to a FINRA trade reporting facility until October 2021—several months after it apparently commenced fractional share trading.

This second challenge is greatly diminished, however, by the fact that unlike the consolidated tape data, the OTC Transparency data can be back-filled to correct errors, and both Robinhood and Drivewealth are in the process of a *post facto* correction to their OTC Transparency data. Our analysis below indicates that this process has been largely completed by Robinhood as of this writing, whereas the process for Drivewealth remains on-going. Therefore, we exercise caution when we encounter a stock-week in the OTC Transparency data that lacks an entry for Drivewealth trades, as the absence of trades does not necessarily mean that Drivewealth executed no fractional share trades in that security. We additionally show below how our classification rules can be used to validate whether the OTC Transparency data has been fully updated.

Notwithstanding these limitations, the OTC Transparency data provide novel insight into the growth of fractional share trading in today's equity market. The FINRA data separates trades made by ATS venues and other off exchange trading (or what FINRA designates as "OTC (Non-ATS).") This latter category includes internalized trades by retail market makers. Until 2019, the non-ATS category was dominated by retail market makers such as Citadel Securities and Virtu. Starting in 2019 and 2020, however, the composition of firms included in these disclosures included retail brokerage firms as they began fractional share trading programs. For instance, in Figure 1, we plot the natural log of the aggregate weekly number of trades disclosed by Citadel Securities LLC and Virtu Americas LLC between January 1, 2019 and March 31, 2022, as well as the top six retail brokerage firms by trades reported in March 2022. Review of the customer account agreements for each of these brokerage firms confirms that each adopted the second approach to fractional share trading discussed previously, whereby the fractional portion of each trade is required to be disclosed to FINIRA as a whole share trade. That each brokerage firm appears in the OTC data shortly after it began offering fractional share trading further indicates that these trades are likely to reflect fractional share executions.

[Insert Figure 1]

Examining the average trade size for trades reported by these retail brokerage firms similarly provides additional evidence that these reported trades primarily reflect fractional share trades. In Table 1, we present the aggregate number of trades and shares reported by each firm shown in Figure 1 for the same time period. While both Citadel and Virtu had an average trade size across all reported trades of over 300 shares per trade, the average was close to 1.0 for each retail brokerage firm.

[Insert Table 1]

We next use these disclosures to build our classification rules for identifying fractional share trades in the TAQ data.

C. *Identifying Fractional Share Trades in the TAQ Data*

We exploit the large number of fractional share trades executed by Robinhood and Drivewealth since 2021 to build a classification rule for identifying these trades for each firm in the TAQ data. For brevity, we describe the method by which we build our classification rules in Appendix A and focus here on providing a general overview of these rules, along with an assessment of their accuracy.

In general, each rule is based on the fact that the trade reports for the fractional share trades of Robinhood and Drivewealth that appear in the TAQ data should have a number of distinctive characteristics. For instance, as we show in Appendix A, these trades should appear as single share executions in one of three FINRA trade reporting facilities (TRFs). More importantly, each trade report reflects the time it takes for a broker to report the trade to one of two Securities Information Processors, and fractional share trades executed by both Robinhood and Drivewealth have distinctively long trade reporting latencies. As such, by examining the tail of the distribution of reporting latencies for single share FINRA trades before and after these firms commenced reporting trades to the tape, we estimate the latency distributions for fractional share trades executed by each firm.

To assess the accuracy of the classification rules, we turn to the OTC Transparency data. In conventional predictive modelling, classifiers such as the ones we construct would be validated against a test dataset of trades that are known to have the target characteristic of interest (e.g., RH=0/1). While we lack access to such a dataset, the OTC Transparency data provides a useful substitute for those stock-weeks where Robinhood or Drivewealth disclosed trades. In particular, if the OTC Transparency data reveal that either Robinhood or Drivewealth executed only single share trades in a particular security during a trading week, the firm's weekly trades in that security likely reflect fractional share trade executions. We therefore evaluate the extent to which applying our classifiers to the TAQ trade data results in weekly trade estimates for Robinhood and Drivewealth for a stock that match their disclosures for these stock-weeks in the OTC Transparency data.

In Figure 2, we illustrate this approach using trades in Tesla (TSLA). Trades in TSLA were the most common trades disclosed in the OTC Transparency data for both brokers, with Robinhood reporting over 14 million total trades between January 1, 2020 and the week of March 28, 2022 and Drivewealth reporting over 8 million. Moreover, non-fractional single share FINRA trades are common in TSLA. For instance, during 2020—a year when presumably few, if any, fractional share trades were reported as single share trades—there were approximately 75,000 single share, non-exchange trades in TSLA on any given trading day. The large number of non-fractional, single share FINRA trades thus complicates identifying those single share FINRA trades that are actually fractional share executions.

As shown in Figure 2, however, our classification rules perform well even in this trading environment. After Robinhood began reporting fractional share trades to the tape in mid-February 2021, the weekly estimate for Robinhood fractional share trades in Panel A of Figure 2 is roughly 93% of the weekly numbers disclosed in the OTC Transparency data. In Panel B, the estimates for Drivewealth's fractional share trades after October 6, 2021—the date Drivewealth appears to have begun reporting trades to the tape—are even more closely aligned to those disclosed for Drivewealth in the OTC Transparency data. However, whereas the estimates for Robinhood's trades are slightly below the OTC Transparency figures, the estimates for Drivewealth's are slightly higher. Specifically, our Drivewealth estimates exceed the disclosures in the OTC Transparency data by an average of 6% each week. In the Appendix, we explain why the classification rule for Robinhood is likely to underestimate slightly the actual number of fractional share trades executed by Robinhood, while it is likely to overestimate slightly the number for Drivewealth.

[Insert Figure 2]

We additionally examine the cross-sectional performance of our two classifiers for all stock-weeks in the OTC Transparency Data where the reported trades and reported shares traded were the same number for either Robinhood or Drivewealth. Given that Robinhood's fractional share trades do not appear in the TAQ data until February 16, 2021, we confine our Robinhood analysis to the week of March 1, 2021 through the week of March 28, 2022. For Drivewealth, we similarly use the weeks from November 1, 2021 through March 28, 2022 to ensure that we capture TAQ data during a time when Drivewealth was reporting fractional share trades to the tape. With these restrictions, our data consist of 14,262 stock-weeks for Robinhood and 4,913 stock-weeks for Drivewealth.

As shown in Figure 3, both classifiers perform well in estimating the number of trades reported by Robinhood and Drivewealth in the OTC Transparency data. In Panel A of Figure 3, we present a simple scatter plot of the number of weekly trades reported for Robinhood in the OTC Transparency data for each stock-week against the weekly estimate of these trades after applying our Robinhood classifier to the TAQ trade data. Panel B presents the same figure for Drivewealth. Overall, both panels indicate a nearly one-for-one correspondence between the weekly estimate of trades and disclosed trades for both firms.

[Insert Figure 3]

In Table 2, we estimate this relationship directly by regressing the weekly trades disclosed for each stock-week on the weekly estimates for each firm. Column 1 presents the estimates for Robinhood, while Column 2 presents the estimates for Drivewealth. For Robinhood, the regression yields a precisely estimated coefficient of 1.068, consistent with our observation above that our Robinhood classifier slightly underestimates the disclosed number of Robinhood trades. For Drivewealth, the regression likewise yields a precisely estimated coefficient of 0.926, indicating that, on average, our Drivewealth classifier slightly overestimates the number of disclosed trades for the firm. In both cases, our two classification rules capture almost all of the cross-sectional variation in fractional share trading by Robinhood and Drivewealth, as reflected in the very high R-squared of 0.991 and 0.995, respectively.

[Insert Table 2]

3. Who Uses Fractional Shares? Fractional Share Trades in the Cross Section

A natural question to ask is whether fractional share trades matter. If retail traders use fractional share trades primarily to purchase a fraction of out-of-reach, high-priced stocks, the incidence of fractional share trades should be confined to only a small portion of U.S. equity securities. On the other hand, the emergence of fractional share trading has also enabled retail brokerage firms to offer investors the ability to enter orders based on the dollar value of the trade rather than the number of shares to be traded. Indeed, the default method for entering trades on both Robinhood and Cash-App (a Drivewealth application) is to use dollars and cents rather than shares, and as noted, many applications that rely on Drivewealth (e.g., Revolut) specifically allow users to "round up" credit card transactions to invest the rounded-up portion in stocks. To the extent investors enter orders based on their dollar value, the resulting trade will likely involve a fractional share component, increasing the incidence of fractional share trades across stocks of all prices. More generally, this latter form of trading would be indicative of a particular type of retail order flow—i.e., trades originating from a mobile-based application entered in dollar values—that may differ from broader measures of retail order flow.

To assess the prevalence of fractional share trading, we focus our analysis on trading in the common stock of all U.S. firms contained in the daily stock file maintained by the Center for Research on Securities Prices (CRSP). Because neither Robinhood nor Drivewealth reported fractional share trades to the tape prior to February 15, 2021, our sample consists of all trading days between March 1, 2021 and March 31, 2022. After filtering for securities having a share code of "11" (representing common equity of a U.S. corporate issuer), the sample consists of 1,160,627 stock-days, comprised of 4,648 stocks issued by 4,601 firms. For each issuer, we also collect bi-monthly short interest data using the CRSP/COMPUSTAT merged file. Lastly, we merge onto each stock-day the total number of Robinhood trades and Drivewealth trades observed for the stock-day within the TAQ data based on the classification rules described in Section 2.[15]

In Table 3, we present a list of the 50 securities with the largest number of fractional share trades executed by Robinhood and Drivewealth. As noted, TSLA represented the stock with the largest number of RHDW fractional share trades during our sample period. Column (5) indicates

[15] Efforts to match stocks-days from CRSP to stock-days in TAQ using the Daily TAQ CRSP Link available through CRSP resulted in an incomplete match of stock-days between these data. Often, this was due to the fact that while stock symbols often change due to corporate reorganizations and mergers, the PERMNO associated with a company's equity securities in CRSP typically remains the same. Where the link file failed to produce a confirmed match, we matched PERMNO-DATE observations in CRSP to SYM_ROOT-SYM_SUFFIX-DATE observations in TAQ by means of hand-matching combinations. Through this method, we match 100% of the 1,160,627 PERMNO-DATES to a SYM_ROOT-SYM_SUFFIX-DATE observation in TAQ. Specifically, the 4,648 PERMNOs observed in our CRSP data were associated with 4830 SYM_ROOT-SYM_SUFFIX observations in our TAQ data.

that during the sample period roughly 6.7% of all trades in Tesla were fractional share executions by Robinhood or Drivewealth, based on our classification rules. Moreover, Columns (7) and (8) underscore the importance of fractional share trading in understanding the large number of single share trades now reported for most issuers. In the case of TSLA, fractional share executions by Robinhood and Drivewealth accounted for over 25% of all single share trades in the market, and a remarkable 48% of single share trades reported to FINRA. Nor is this phenomenon limited to Tesla; across all 50 stocks in Table 3, fractional share trades accounted for between 11% to 40% of all reported single share trades and between 27% and 51% of all reported FINRA trades. We return to this issue below in Section 5 when we discuss the distortions to the consolidated tape caused by the current rules for fractional share trade reporting.

[Insert Table 3]

More generally, the high ranking of TSLA in Table 3 suggests fractional share trading should be especially common in high-priced stocks that are popular among retail traders. As shown in column (1), Tesla had an average stock price of over $800 during the sample period. Likewise, using data from 2018 through August 2020 provided by Robintrack.net (which tracked the number of Robinhood accounts holding a particular stock), Welch (2022) finds that Tesla was a common stock held by Robinhood investors.

Yet Table 3 also reveals that fractional share trading was not confined to the most expensive stocks. For instance, the second stock on the list is AMC Entertainment, an especially noteworthy "meme" stock whose stock price ranges between $9 per share and $59 per share during the sample period: in other words, not expensive. Overall, Table 3 includes just five stocks (Tesla, Amazon, Netflix, and both classes of Alphabet C) with an average stock price of $500 or more during the sample period. The remainder of the list is largely dominated by "household" companies (such as Apple, Disney, Nike, Coca Cola Co, and Walmart), other "meme" stocks (such as Gamestop), and several recent IPOs (such as Rivian and Coinbase) and Special Purpose Acquisition Companies (SPACs) (such as Digital World Acquisition Corp., Draftkings Inc., Virgin Galactic Holdings, Inc., Lucid Group Inc., SoFi Technologies, and ChargePoint Holdings) that were reportedly popular among retail traders.[16] Moreover, most of

[16] See, e.g, Isabelle Lee, "SPACs are booming 'at the expense of retail investors', and regulators should take these 5 steps to fix the market, think tanks say", *Business Insider*, March 7, 2021, available at

the stocks on Table 3 had stock prices during the sample period that were just a fraction of the price of Tesla. For example, twenty had an average stock price of less than $50 per share.

Table 3 also indicates that fractional share trades are far from uniformly distributed across stocks. As shown in the last three rows of the table, fractional share trades in these fifty stocks accounted for 89.3 million of the 193.1 million fractional share trades (46%) estimated to have been executed by Robinhood and Drivewealth based on our classification rules. In Table 4, we expand this analysis to stocks beyond the "top 50" based on the ranking of stocks by the total number of fractional share trades. As shown in column (1), including the top 100 stocks captures nearly 60% of all estimated fractional share trades, while expanding the list of stocks to the top 500 captures nearly 80% of all trades. Even though our sample includes over 4,600 stocks, Table 4 indicates that focusing on just slightly less than half of the sample stocks would nevertheless capture almost 95% of all estimated fractional share trades. For comparison, we also calculate the number of total retail trades observed for each stock in the sample during the sample period based on the BJZZ metric and similarly rank stocks from highest to lowest by number of estimated retail trades. While the distribution of retail trades based on this metric is also skewed toward the top-ranked stocks, fractional share trades are even more concentrated among just a fraction of all stocks within the CRSP dataset. For instance, as shown in columns (2) and (3), where the "top 50" stocks in the RHDW ranking comprise roughly 47% of all observed RHDW fractional share trades, the top 50 stocks in the retail trade ranking comprise only 33% of observed retail trades. Likewise, for the top 100, the figures are 57.26% versus 43.98%, and for the top 200, they are 67.43% versus 55.65%.

[Insert Table 4]

What might induce investors to concentrate fractional share trades within such a small subset of U.S. equity securities? Somewhat surprisingly, column (5) of Table 3 indicates that a high stock price appears to be only a partial explanation. While over 10% of all trades in Amazon (average stock price=$3,293.80) are fractional share executions, over 8% of trades in Gamestop were also fractional share executions despite having an average stock price that was roughly 1/20th that of Amazon.

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https://markets.businessinsider.com/news/stocks/spac-boom-sec-retail-investors-blank-check-companies-regulation-2021-3-1030157257.

In Figure 4, we plot the average percent of trades in a security that are fractional share executions within the sample against a stock's average daily price. Due to skewness in both measures, we plot the relationship using the natural log of both measures. The figure has a number of distinctive features. First, the outlier in the top right quadrant represents the Class A common stock of Berkshire Hathaway (BRK.A), which had an average price of approximately $425,000 per share during our sample period. As shown in Bartlett, McCrary and O'Hara (2022), fractional share trades by Robinhood or Drivewealth now represent roughly 80% of all trades in BRK.A, underscoring the critical role that fractional share trades now play in the average daily volume of the market's most expensive publicly-traded stock. However, the figure also reveals that the relationship between the incidence of fractional share trading and stock price is hardly straight-forward. Excluding BRK.A, the slope coefficient on a regression line through these data is -0.02 (robust std. error=0.009). Indeed, the distinct, vertical cluster of firms with an average closing price of roughly $10/share (natural log=$2.30) indicates that stock price may often be orthogonal to fractional share trading. This vertical cluster of stocks largely reflects the large number of SPACs launched during 2020 and 2021. SPACs raise capital in an IPO and thereafter search for an acquisition target. Notably, these issuers are typically structured to trade at $10/share after their IPO until they acquire a firm, and they structure any subsequent acquisition to be completed at a stock price that values the SPAC's securities at $10/share. While many of these stocks had very little fractional share trades, others ranked as among the stocks with the highest percentage of RHDW fractional share trades in our sample. Likewise, our sample also coincided with a large number of conventional IPOs that were priced at less than $20/share, but their trading was associated with a high frequency of fractional share trades. Overall, among the 80 stocks where RHDW fractional share trades constituted more than 5% of all trades, nearly half (N=36) had an average closing price of less than $20/share.

[Insert Figure 4]

Aside from BRK.A, the absence of any clear, positive relationship between stock price and RHDW fractional share trades in Figure 4 is consistent with RHDW fractional share trades arising from Robinhood and Drivewealth investors commonly entering trades for an overall dollar value (e.g., invest $1,000 total across these 3 stocks). To explore which types of stocks are likely to be of interest to this class of retail investors, we use our sample to estimate the following model:

$$RHDW_{it} = \alpha_{it} + retail\_flow_{it} + price_{it} + mktcap_{it} + new_{it} + age_{it} + volatility_{it} + turnover_{it} + short\_interest_{it} + tape_{it} + \varepsilon_i \quad (1)$$

Our outcome variable $RHDW_{it}$ is measured as the natural log of the number of RHDW fractional share trades in stock *i* observed on day *t.* Our interest is in why retail traders at Robinhood and Drivewealth use fractional share trades above and beyond the fact that a particular stock might be popular among retail investors. We therefore control for overall retail order flow within our sample by including the covariate $retail\_flow_{it},$ which is an estimate of retail trades in stock *i* on day *t* that are executed by retail market makers using the BJZZ metric. This proxy for retail trades is based on non-exchange trades having sub-penny price improvement, exclusive of midpoint executions. As we show below, fractional share trades executed by Robinhood are overwhelmingly at the midpoint, and those executed by Drivewealth are overwhelmingly executed with no price improvement. The BJZZ metric, therefore. does not include the vast majority of the fractional share executions in our sample.

By controlling for retail order flow, the model establishes the following (sharp) null hypothesis for the remaining covariates: If fractional share trades were randomly distributed across retail trades, none of the other covariates should be statistically associated with fractional share trades. These covariates are (for each stock *i* on day *t*) the natural log of the closing price ($price_{it}$), the natural log of market capitalization ($mktcap_{it}$), an indicator for whether the stock has been traded for less than 180 calendar days (new_{it}), the natural log of the number of days the stock has traded (age_{it}), the natural log of return volatility over the prior 30 days ($volatility_{it}$), the natural log of the volume of shares traded relative to total shares outstanding ($turnover_{it}$), the natural log of the most recent bi-monthly short-interest ratio ($short\_interest_{it}$), and an indicator for stock exchange listing ($tape_{it}$).

Estimates from this model are presented in Table 5. Column (1) provides estimates for the baseline model, while column (2) estimates the same model with date fixed effects. For both models, the log-log specification permits coefficient estimates to be interpreted as the elasticity of RHDW fractional share trades with respect to the variable in question. As expected, the tightly estimated coefficients for *retail flow* indicate that RHDW fractional share trades generally track retail order flow; in particular, a 10% increase in the BJZZ proxy for retail order flow is associated with a 6% increase in RHDW fractional share trades. Likewise, after controlling for retail order flow, the model reveals that a 10% increase in stock price is associated with a roughly 2% increase in RHDW fractional share trades. Similarly, the model indicates that fractional share trades are especially common among larger capitalization companies, as well as newly public companies. Indeed, even among companies that have traded for more than six months, the model indicates a negative relationship between fractional share trades and the length of time a company has publicly traded. Consistent with Welch (2022), RHDW fractional share trades are also increasing in return volatility and trading volume. Lastly, columns (1) and (2) indicate that RHDW fractional share trades are more common among Nasdaq-listed firms, as well as among stocks with higher levels of short interest, suggesting skepticism among short sellers about the stock's value.

In column (3), we add stock fixed effects to examine how the daily changes in the level of RHDW fractional share trades within a stock were associated with changes in the model's time-varying covariates. These within-stock estimates are largely consistent with the between-stock estimates for wholesale retail order flow, price, market capitalization, volatility, and turnover. However, the coefficient signs are the opposite for a stock's age and short-interest, indicating that for any given stock, fractional share trading grows with a stock's age but is decreasing in the stock's level of short interest.

[Insert Table 5]

Overall, the estimates from Table 5 suggest that investors who choose to execute fractional shares on the Robinhood and Drivewealth applications generally trade stocks that are both popular among retail traders in general but also those that have particular attributes. Specifically, their trading selection skews toward those stocks with higher market capitalizations and recent trading volumes, as well as those that are newer and more volatile.

4. **Do Fractional Share Trades Contain Information?**

In this section, we explore the question of whether fractional share trades, despite their tiny size, nevertheless contain unique, value-relevant information.

A. Motivation

There are two primary reasons why fractional share trade data may contain valuable information for traders. First, a large number of fractional share trades arise from Robinhood investors, and using data from Robintrack prior to its de-commissioning in August 2020, several studies have found that Robinhood investors possess distinct characteristics and that Robinhood trading is associated with future market conditions. For instance, Welch (2022) finds that Robinhood investors aggressively purchased stocks during the initial pandemic downturn and after the ensuing episodic market upswings, consistent with uninformed trading. Similarly, Barber et al. (2021) find that, relative to other retail traders, Robinhood investors engaged in more attention-induced trading and that intense trading by Robinhood investors is associated with negative future returns. Eaton, Green, Roseman, and Wu (2022) likewise find that decreases in Robinhood investor trading due to platform outages are associated with higher market liquidity, suggesting that Robinhood investor trading harms liquidity. While these studies focused on Robinhood investors, it is plausible that investors using other app-based trading platforms (e.g., those powered by Drivewealth) may share similar characteristics to Robinhood investors.

Second, from an empirical perspective, the prevailing measure of retail order flow—the BJZZ metric—is likely to capture this form of uniformed order flow with considerable noise. By focusing on FINRA trades with sub-penny price improvement (excluding those priced at the midpoint of the NBBO), the BJZZ metric seeks to identify internalized retail trades, regardless of the broker. Consequently, the measure captures trades from conventional retail brokers as well as newer app-based platforms. In addition, as noted by Shearer (2022), the BJZZ also captures non-retail, institutional trades executed through single dealer platforms (SDPs), which (like retail market makers) internalize trades for institutional clients and frequently provide sub-penny price improvement. To the extent fractional share trades originate from investors using trading apps to place dollar-priced orders, they therefore provide a cleaner metric for the type of retail, app-based trades that existing studies have found to be associated with future market conditions.

Before proceeding, however, we make two caveats regarding our analysis. The first is with respect to causation. The question we address is whether fractional share trading provides information with respect to future market conditions, regardless of why this might be the case. As such, while much of the literature on retail trading seeks to estimate the causal effect of retail trading on prices and liquidity, we expressly refrain from making any such claims here. For instance, we take no position on whether fractional share trading causes a stock to become more (or less) volatile or whether traders who use fractional share executions are simply drawn to stocks that are likely to become more (or less) volatile.

The second caveat is with respect to an important limitation of the RHDW fractional share trade data for purposes of examining its informational content. Specifically, examination of the trade price relative to the TAQ NBBO at the time of a RHDW fractional share trade cautions against making inferences from the order imbalance between trades that the TAQ NBBO indicates might have originated from marketable buy orders relative to marketable sell orders. For instance, in Table 6, we present a number of trade execution statistics for our sample of fractional share trades, presented separately for Robinhood and Drivewealth by the time of the day. To ensure comparability between Robinhood and Drivewealth trades, the sample period is restricted to November 1, 2021 through March 31, 2022 when both firms were reporting fractional share trades to the tape.

[Insert Table 6]

Turning first to Robinhood, across the roughly 51 million trades classified as Robinhood fractional share executions during this time period, roughly 20% occur within the first thirty minutes of the trading day, while 75% occur between 10:00 am and the end of the regular trading session. However, the rates of buys, sells, and midpoint executions (based on the TAQ NBBO prevailing at the time of the trade) across these two time periods appear to be quite different. For instance, while 75% of trades occur at the midpoint between 10:00 am the close of the regular session, the figure is just 55% for trades between 9:30 am and 10:00 am. Moreover, over 90% of trades occurring outside the regular trading session, when liquidity is scarce, are executed at the midpoint. Visual examination of the data indicates that Robinhood's standard practice is to price fractional share executions at the midpoint and that the basis for these differentials is simply due to differences in the TAQ NBBO and the NBBO utilized by Robinhood for pricing. For example, examination of trades identified as "buy" or "sell" orders between 9:30 am and 10:00 am are priced in decimals ending in either two decimals or a half-penny, consistent with midpoint pricing.[17] In light of this evidence, we therefore assume Robinhood seeks to execute all trades at the midpoint and do not formally assign trade direction to Robinhood trades.

In contrast, assigning trade direction to Drivewealth fractional share trades using the TAQ NBBO does not suffer from these concerns, but it suggests caution for other reasons. As with Robinhood, Drivewealth executes a substantial portion of its fractional share trades (38%) during the first thirty minutes of the regular trading session, but in contrast to Robinhood, it prices very few trades at the NBBO midpoint. On the contrary, most trades appear to be executed at precisely the NBBO with no price improvement. Between 9:30 a.m. and 10:00 a.m., nearly 60% of trades are priced in this fashion—a number that increases to over 73% between 10:00 am and the close of the regular session. Yet while assigning trade direction for Drivewealth trades based on the TAQ NBBO appears to rest on a sounder footing, Table 6 also indicates a clear bias in favor of buy orders.[18] While we are unaware of the precise reason for this bias, it is consistent with the "round up" feature offered through Drivewealth.[19] For instance, Cash-App utilizes Drivewealth to execute fractional share trades, and it also offers a Cash-App debit card that allows users to "round up" purchases to buy fractions of a share.[20] (For example, after buying a coffee for $2.75, the Cash-App user might allocate $0.25 to a purchase of a pre-specified security.) To the extent a user elects to deploy this feature, the user would execute multiple sub-dollar buy transactions as they utilize the debit card, thus inflating the number of buy transactions relative to sell transactions within the Drivewealth trade data. Regardless of whether this is the precise reason for the bias, the stark disparity between buy and sell orders within the Drivewealth data suggests the need for caution.

In light of these concerns, we do not assign trade direction to any RHDW fractional share trades and focus instead on their overall frequency within the data.

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[17] Likewise, these "open" trades are also frequently printed in clusters that have prices reflecting recent but stale midpoint prices, which may reflect challenges that Robinhood faces in processing the large volume of orders it receives prior to the market opening.

[18] This is true for all periods of time other than the 0.002% of trades that occurred after the regular trading session.

[19] https://www.drivewealth.com/solutions/round-up/

[20] https://cash.app/help/us/en-us/10131-cash-card-round-ups

B. *Estimation*

Our empirical test examines whether RHDW fractional share trading is predictive of a stock's future liquidity and volatility. In this regard, our framework is similar to the one adopted in Jones, Zhang and Zhang (2022). Using the BJZZ measure of retail trading, these researchers find that between January 2020 and March 2022, higher daily retail trading in a stock is associated with higher next-day effective spreads and volatility. These findings stand in contrast to prior research showing that retail trading enhances liquidity and lowers volatility (see, e.g., Barrot, Kaniel, and Sraer (2016) and Foucault, Sraer, and Thesmar (2011)). Jones et al. suggest the reason may stem from the evolution of retail trading following the emergence of commission-free trading platforms, the rise of social media as information gathering and distribution channels, and the entry of "Robinhood-type" investors. Yet this interpretation of their results is complicated by virtue of the BJZZ metric for the reasons discussed previously.

If the association between retail trading and future liquidity and volatility is driven by "Robinhood-type" investors, we hypothesize that this association should be stronger when we estimate the association using RHDW fractional share data relative to when it is estimated using the BJZZ metric. Using the same sample of stock-days described in Section 3, we test this hypothesis by means of a series of "horserace" regressions based on the following baseline regression specification:

$$Y_{i,t} = \alpha_0 + \beta_1 Retail_{i,t-1} + \beta_2 Fractional_{i,t-1} + \beta' X_{i,t-1} + \delta_i + \gamma_t + \varepsilon_{i,t} \quad (2)$$

The outcome of interest, $Y_{i,t}$, represents one of three different liquidity and volatility measures for stock *i* on day *t*. For liquidity, we use the Simple Averaged Percentage Effective Spread provided in the WRDS Intraday Indicators.[21] For volatility, we examine two measures. The first volatility measure is a stock's trade-based intraday volatility which is also provided in the WRDS Intraday Indicators.[22] The second is the weighted-average implied volatility across all

[21] This measure is calculated using the daily TAQ data across all trades for each stock *i* on day *t* as:

$$\frac{1}{N} * \sum_{k=1}^{n} \frac{2D_k(P_k - M_k)}{M_k}$$

where *N* is the number of observed trades, D_k is 1 if the trade is signed as a buy order and -1 if signed as a sell order based on Lee and Ready (1991), P_k is the trade price, and M_k is the NBBO midpoint at the time of the trade.

[22] This measure is calculated using the daily TAQ trade data as the return volatility across all observed trades for stock *i* on day *t*.

call options on stock *i* on day *t* based on the historical option pricing data provided in OptionMetrics.[23] The covariates of interest are $Retail_{i,t-1}$, which represents the natural log of the total dollar volume of retail trades in stock *i* on day *t-1* based on the BJZZ metric, and $Fractional_{i,t-1}$, which represents the natural log of the total number of RHDW trades on the same stock-day. Following Jones et. al. (2022), the regression also includes (for each stock i on day t-1) a vector of control variables, X, that the prior literature suggests can affect liquidity and volatility. These include three measures of prior stock returns (prior day returns, prior week returns, and prior month returns), the prior month's return volatility, as well as the natural log of the following measures: the dollar volume of trades, the total number of trades, market capitalization, turnover (measured as the previous month's trading volume scaled by outstanding shares), and the book-to-market ratio. All regressions are run with firm (δ_i) and time (γ_t) fixed effects, and we additionally include the lagged dependent variable in all regressions to control for time series persistence in our outcome measure. Standard errors are double-clustered by day and stock.

As described in Section 3, our primary sample consists of 1,160,627 stock-days from March 1, 2021 through March 31, 2022, which represents our sample period for our estimates of effective spreads and intraday volatility. For estimating implied volatility, our sample period ends on December 31, 2021 as data for OptionMetrics are not available beyond this date as of this writing.

For all three outcome measures, our sample represents the universe of common equity issued by U.S. issuers within the CRSP data to the extent matches can be found in COMPUSTAT and, for models of implied volatility, OptionMetrics. Yet Section 3 also highlighted that RHDW fractional share trades are highly concentrated in select stocks, with nearly 60% of all RHDW fractional share trades occurring in just 100 stocks. To the extent RHDW fractional share trading is predictive of future liquidity and volatility, the effect should be especially pronounced among the subset stocks with high levels of fractional share trading. We therefore supplement our analysis by estimating the following, modified version of Equation 2 to examine this possibility:

[23] The weighted average was calculated using the daily trading volume for each option contract on stock *i* on day *t*.

$$Y_{i,t} = \alpha_0 + \beta_1 Retail_{i,t-1} + \beta_2 Fractional_{i,t-1} + \beta_3 Top100_{i,t-1} * \ Retail_{i,t-1} + \beta_4 Top100_{i,t-1} * \ Fractional_{i,t-1} + \beta' X_{i,t-1} + \delta_i + \gamma_t + \varepsilon_{i,t} \quad (3)$$

$Top100_{i,t-1}$represents an indicator for whether stock *i* ranks among the top 100 stocks in terms of total RHDW fractional share trades between March 1, 2021 and March 31, 2022. By interacting this term with $Retail_{i,t-1}$ and $Fractional_{i,t-1}$, the coefficients β_3 and β_4 estimate the degree to which the association between these two measures differs for stocks with especially high levels of fractional share trading.

Regression results are presented in Table 7. For each outcome, we first estimate Equation (2) separately for $Retail_{i,t-1}$ and $Fractional_{i,t-1}$ before including them together in the model. For instance, column (1) estimates the association between effective spreads and daily retail trading volume without including $Fractional_{i,t-1}$ as a covariate, while column (2) does the opposite. Consistent with Jones et al., the coefficient for *Retail* is positive, suggesting next-day effective spreads are increasing in today's daily retail trading, but the result is not statistically significant during our sample period. In contrast, the positive coefficient for *Fractional* in column (2) is over seven times as large in magnitude and has a t-ratio of more than 9. This result is unchanged in column (3) when we include both measures as covariates. As these covariates and all outcome measures are in logs, the regression estimates indicate that a 10% increase in RHDW fractional share trades is associated with an 8 basis point increase in the following day's Averaged Percentage Effective Spread. Note, however, that to the extent there is variation in daily RHDW fractional share trades, it is quite often far in excess of a 10% change. For instance, within the sample, the average daily change in the total number of RHDW trades has a mean of 114% and a median of 43%. Over one-quarter of daily changes are in excess of 82%.

[Insert Table 7]

Column (4) presents the modified model with interaction effects. The coefficient for *Fractional* remains largely the same as in columns (2) and (3), but the interaction of *Top100* and *Fractional* yields a coefficient estimate of 0.0667, which is over 9 times as large as the estimate for *Fractional* alone. In contrast, the interaction of *Top100* and *Retail* is -0.0255 with a t-ratio of over 3 in magnitude. Overall, for stocks with large levels of fractional share trading, these estimates suggest that, as in Barrot, Kaniel, and Sraer (2016) and Foucault, Sraer, and Thesmar

(2011), conventional retail trades are associated with higher future liquidity, while "Robinhood-type" trades (as proxied by RHDW fractional share trades) are predictive of lower future liquidity.

In columns (5) through (8), we conduct the same analysis for predictors of intraday trade volatility. In contrast to our estimates for effective spreads, the coefficient estimate of 0.00872 for *Retail* in column (5) is now both positive and sufficiently precise to distinguish it from zero. As with effective spreads, however, the elasticity of 0.0157 for *Fractional* in column (6) is nearly twice as large and more precisely estimated. Notably, both estimates remain positive and statistically significant in column (7), although the estimate for *Retail* declines in magnitude by roughly 15% while the estimate for *Fractional* declines by just 3%. In column (8), the interaction effects are again especially noteworthy. While the main effects for *Retail* and *Fractional* remain consistent with the estimates in column (7), the interaction term of *Top100* with *Fractional* yields a tightly estimated elasticity of 0.257 while its interaction with *Retail* yields a similarly precise estimate of -0.0975.

Finally, columns (9) through (12) provide the same set of estimates for implied volatility. Here, the results in columns (9) and (11) continue to indicate that the level of daily RHDW fractional share trading is predictive of the following day's implied volatility. However, coefficient estimates indicates that the estimated elasticity for *Retail* is slightly larger than that for *Fractional* across all three columns. Nevertheless, as with effective spreads and intraday volatility, the interaction effects in column (12) continue to reveal the especially strong association between RHDW fractional share trades and volatility among stocks with high levels of fractional share trading. In particular, the interaction of *Top100* and *Fractional* yields an estimated elasticity of 0.0306 with a t-ratio of 4.44, whereas its interaction with *Retail* yields an estimate of 0.00937 with a t-ratio of just 1.68.

5. Some Not So Tiny Problems with Existing Data on Fractional Share Trades

As the foregoing sections show, the trading data for RHDW fractional share trades provides information on a distinct form of retail order flow that is relevant for predicting liquidity and volatility, but it also suffers from a number of limitations. As noted in Section 2, perhaps the largest limitation is that it reflects only a portion of fractional share trades that occur in the market, given that retail fractional share trades executed by Apex and others are not presently observable in the consolidated tape or in the OTC Transparency data. Additionally, Table 1 indicates that, among brokers that report fractional share trades to the tape, Robinhood and Drivewealth were by far the most active during our sample period; however, Figure 1 underscores that other brokerage firms are quickly gaining ground. Equally important, Robinhood or Drivewealth could alter their trade execution practices such that the classification rules used in this study no longer track their fractional share trades.

The existing reporting regime also raises a more general question about the accuracy of stock trading volume data for all U.S. equity securities due to the "rounding up" rule, as highlighted in Bartlett, McCrary and O'Hara (2022) using trades in BRK.A. In addition to potentially misleading investors, inflated trading volume can directly affect firms and investors due to the importance of trade volume data across a number of U.S. securities regulations. For instance, rules relating to the quantity of stock that may be repurchased by a company or sold by corporate insiders both hinge on the reported trading volume in a firm's securities, as does the ability of a firm to qualify as a foreign private issuer.[24] Reported trading volume is also among the "Cammer factors" that determine whether investors can bring a class action securities fraud lawsuit under Rule 10b-5.[25] In this section, we explore both of these deficiencies of the current trade reporting regime for fractional share trades.

A. Can the OTC Transparency Data Provide a Substitute Source of Data on Fractional Share Trades?

In Section 2, we evaluated our classification rules for RHDW fractional share trades by comparing our estimated weekly tallies of such trades with those disclosed by Robinhood and Drivewealth in the OTC Transparency data. In this regard, one might view these latter data as a possible source of obtaining weekly, aggregate data for fractional share trades executed by those retail brokers that report fractional share trades to a FINRA trade reporting facility. However, as discussed in Section 2, FINRA's disclosure of weekly trades by brokerage firm is subject to a "de minimis" trading threshold: Weekly trades will not be attributed to a specific FINRA member and will instead be aggregated and reported as completed by "De Minimis Firms" if

[24] See, e.g., 17 CFR § 230.144 (providing safe harbor for certain control person resale transactions so long as all trades within the past three months do not exceed the average weekly reported volume of trading for the issuer); 17 CFR § 240.12h-6 (allowing a foreign private issuer to terminate its Exchange Act registration if the U.S. average daily trading volume of the subject class of securities has been no greater than 5 percent of the average daily trading volume of that class of securities on a worldwide basis for a recent 12-month period).

[25] https://corpgov.law.harvard.edu/2012/08/23/do-courts-count-cammer-factors/

either (a) the trades were executed by a FINRA member who executed fewer than 200 transactions per day for the reporting week, or (b) the trades were completed by a FINRA member who executed on average fewer than 200 transactions per day in the particular security during the week. For a 5-day trading week, this requirement effectively means a broker must execute 1,000 or more fractional share trades in a security for its weekly trades to appear individually in the OTC Transparency data.

We examine the extent to which this de minimis rule excludes fractional share trades from a broker's OTC Transparency data. In particular, for each stock in our sample, we compare our weekly estimate for fractional share trades in Robinhood with the weekly disclosures for Robinhood that appear in the OTC Transparency data. As an example, we present the results for fractional share trades in The Clorox Company (CLX) during our sample period. In Figure 5, we plot the weekly estimate of fractional share trades in CLX executed at Robinhood using our classification criteria (red dots) against the weekly trades in CLX disclosed for Robinhood in the OTC Transparency data (blue bars). Of the 57 weeks that comprised our sample period, the OTC Transparency data for Robinhood is missing for 33 of these weeks. Moreover, as shown in the figure, the reason for these missing weeks was the fact that the total number of fractional share trades executed by Robinhood during these weeks was just slightly less than 1,000 trades per week. With regard to the total number of trades, the OTC Transparency data reported a total of 32,091 whereas our classifier estimates there were 62,766. In other words, the OTC Transparency data captured 42% of the weeks where our classifier indicated that there were RH fractional share trades and 51% of the total estimated RH fractional share trades.

[Insert Figure 5]

We conduct a similar exercise for all stocks in our sample to evaluate the comprehensiveness of the OTC Transparency data with respect to both the total weeks observed and total fractional share trades for a stock. Specifically, for each sample stock we calculate the ratio of (a) the total number of weeks (based on the OTC reporting week) where the OTC Transparency data discloses Robinhood as having executed trades in the stock to (b) the number of these weeks where our classifier detects at least 200 RH fractional share trades in the security.[26] Likewise, for each sample stock, we additionally calculate the ratio of (a) the total number of trades disclosed in the OTC Transparency data for Robinhood during our sample period to (b) the total number of RH fractional share trades estimated for the stock based on our classifier. Importantly, for purposes of both calculations we include in the OTC Transparency data all weekly disclosed trades for Robinhood regardless of the number of shares traded that week. Because some of these trades may reflect whole share trades, our calculation of fractional share trades disclosed in the OTC Transparency data are likely to be overstated.

We present histograms for both measures in Figure 6. Panel A provides the results for weeks observed. For 65% of the stocks in our sample, the OTC Transparency data does not provide data for Robinhood's fractional share trades for even a single week during our sample period where our classifier estimates Robinhood executed at least 200 fractional share trades for the week. For 90% of stocks in our sample, the OTC Transparency data includes data for less than half of these weeks. Across the 3,787 stocks where our classifier finds at least one week with at least 200 fractional share trades, the OTC Transparency data provides full weekly coverage for just 150 (4%) of these stocks. Similar results appear in Panel B, which focuses on the percent of total fractional share trades captured by the OTC Transparency data across stocks in our sample.[27] Among the 35% of stocks where the OTC Transparency data discloses at least one week with Robinhood fractional share trades, the total number of trades is often far less than the total number of trades estimated using our RH classifier. Indeed, for just 329 stocks does this number represent at least 95% of the total RH fractional share trades estimated using the classifier.

[Insert Figure 6]

In summary, because of the de minimis rule, the OTC Transparency data can be used to infer fractional share trade volume for only the most popular retail stocks. And as discussed, this conclusion applies only to brokers such as Robinhood who report each fractional trade to the

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[26] We impose a minimum of 200 fractional share trades to account for the possibility that our Robinhood classifier incorrectly classifies a trade as an RH fractional share trade. For instance, after applying the RH classifier to trades in sample stocks during January 2021 (one month prior to when Robinhood commenced disclosing fractional share trades), the mean number of estimated RH fractional share trades for sample stocks is 21. Assuming a 5-day trading week, this estimate indicates that for an average stock our RH classifier might overstate RH fractional share trades by approximately 100 trades per week. We require at least 200 estimated trades to err on the side of caution.
[27] Where our RH classifier estimated fewer total trades than were disclosed in the OTC Transparency data, we set the percent reported to 1.0 to highlight the extent to which the OTC Transparency data appears to provide 100% coverage of Robinhood fractional share trades.

tape. For brokers such as Apex that do not fill fractional share trades on a principal basis, the OTC Transparency data lacks any information at all concerning the incidence of their fractional share executions.

B. How Much Is Trading Volume Inflated by Fractional Share Reporting Protocols?

As noted previously, Bartlett, McCrary and O'Hara (2022) illustrate how the "rounding up" rule for fractional share trades has resulted in phantom trading volume for BRK.A that now represents 80% or more of its daily trading volume. Yet, they also note that "given its unusually high stock price, BRK.A is in many ways simply an exaggerated example of how FINRA's rule along with the rise in fractional share trading creates phantom, non-existent trading volume across all stocks." In this section we examine the extent to which all securities within our sample experienced inflated trading volume due to the rounding-up rule.

The extent to which the rounding-up rule results in inflated trading volume naturally depends on the actual trade value of each fractional share trade that is rounded up to a whole share. Because we cannot observe trade values, we estimate a range of inflation by assuming that each fractional share trade's true dollar value falls under one of two scenarios. Both scenarios rely on Congressional testimony in February 2021 that the median account value for Robinhood's customers is approximately $240. Under the first scenario, we assume that the dollar value of each fractional share trade is equal to the lesser of $240 and 10% of the reported trade price. Under the second scenario, we assume that its value is equal to the lesser of $240 and 90% of the reported trade price. For example, a fractional share trade that is reported as a whole share trade worth $100 would have an assumed dollar value of $10 under the "10% Scenario" and $90 under the "90% Scenario". This trade would appear in the trade data as a whole share trade; therefore, based on the stock's closing price, our two estimates of "dollar inflation" for this trade would be $90 and $10, respectively. Likewise, our estimates of "percent inflation" would be 90% (i.e., $90/$100) and 10% (i.e., $10/$100), respectively.

Our sample consists of 13.15 billion trades, of which we classify 193 million (1.47%) as fractional share trades executed by Robinhood or Drivewealth. Under the 10% Scenario, these fractional share trades would produce $223.7 billion of dollar inflation due to the rounding-up rule over the sample period. Under the 90% Scenario, this estimate would be $209.1 billion. The total reported trading volume for all trades in the sample is $99.3 trillion based on intra-day TAQ data; therefore, these estimates indicate that total reported dollar volume of trades during our sample period is inflated by 21 to 23 basis points due to the rounding-up rule.

These aggregate figures, however, mask considerable heterogeneity in inflation among stocks. In Table 8, we rank the top twenty stocks by both measures of dollar inflation. Consistent with Bartlett, McCrary and O'Hara (2022), the rounding-up rule was especially consequential for BRK.A, whose dollar volume of trading was inflated by a total of $172 billion under either inflation estimate during the sample period. By itself, this figure accounts for roughly 80% of the total dollar inflation estimate for the sample as a whole. Not surprisingly, the remaining twenty stocks were also among the market's most expensive stocks and accounted for estimated dollar inflation of approximately $42 billion under the 10% Scenario and $34.5 billion under the 90% Scenario. Combined with BRK.A, these amounts represented nearly 95% of the aggregate dollar inflation in the sample.

[Insert Table 8]

Additionally, Table 8 indicates that large dollar values of inflated trading volume are most likely to occur with higher-priced stocks, but it also suggests that the distortion for any given stock will also be a function of its overall liquidity. For example, the rounding-up rule appears to have produced billions of dollars of inflated trading volume during our sample period for Amazon, Tesla, and Alphabet, but these stocks are also among the most heavily traded securities such that their overall dollar volume of trades during the sample was inflated by less than 1%. In contrast, trades in BRK.A typically averaged just a few hundred per day prior to the date that Robinhood commenced reporting trades to the tape. As a result, when Robinhood commenced reporting fractional share trades to the tape—which during our sample period were approximately 1,400 per day—the rounding-up rule meant that roughly 81% of the dollar volume of trades was inflated each day.

Within our sample, we observe many stocks with sparse trading, raising the possibility that a small number of fractional share trades on any given day might distort the reported daily volume for these stocks, even if the aggregate dollar value of the distortion is modest. In Figure 7, we present histograms for the natural log of the average daily percent inflation across all stock-days in the sample using both the 10% Scenario and the 90% Scenario. The natural log of 1% is -4.6; therefore, Figure 7 indicates that the rounding-up rule is unlikely to distort the trading volume for the vast majority of stock-days. Nonetheless, the figure also reveals a nontrivial number of stock-days in the right tail of the two distributions, particularly in the distribution for the 10% Scenario.

[Insert Figure 7]

In Table 9, we examine more closely the stock-days appearing in the tail of these distributions, excluding for this purpose BRK.A. For each stock-day having more than 1% inflation under either metric, we assign it to a five-percent inflation band, from the lowest band of 1% to 5% through the highest band of 85% to 90%. Percent inflation estimates for securities with low stock prices will be more sensitive to assumptions regarding the true value of the trade; therefore, we additionally classify a stock-day by the quintile of its average closing price (based on the average closing prices across all stock-days in the full sample). Each stock-day in the tail of the distributions for Figure 7 is thus assigned to a Percent Inflation Range X Price Quintile bin.

[Insert Table 9]

Columns (1) through (5) of Table 9 present the frequency of stock-days for each bin for the 10% Scenario. For each bin, we also present in parentheses the average number of all daily trades for the stock-days assigned to the bin. Focusing first on column (1), 45 stock-days assigned to the first quintile (stock price<$6.73) had a percent inflation of between 1% and 5%, but only 3 stock-days in this quintile had percent inflation that was higher than 5%, and none had percent inflation that exceeded 20%. In contrast, column (2) reveals that 2,282 stock-days had a percent inflation that exceeded 1%, with 1,171 stock-days having a percent inflation of 5% or more. Remarkably, even though stock-days in the second quintile had an average closing price that ranged from just $6.73 to $13.57, nearly 300 of these stock-days had price inflation of more than 50% under the 10% Scenario, and 234 had price inflation of 85% to 90%—a rate of inflation that exceeded even that of BRK.A.

As with BRK.A, this result stems from the very low level of trading for these stock-days. As shown in column (2), the 234 stock-days with percent inflation of 85% or more had an average of just 40.46 trades for the day. Moreover, examination of the stock-days in column (2) reveals that many of the firms represented pre-acquisition SPACs that were structured to trade at roughly $10/share but also had very modest (and occasionally zero) trades per day as they searched for an acquisition target. As a result, a sudden increase in retail interest in one of these securities could result in sizeable distortions of the stock's daily trading volume to the extent retail investors executed fractional shares trades using Robinhood or Drivewealth. While we cannot rule out the possibility that some of these RHDW trades are false positives under our classification rules, column (2) nevertheless highlights the potential for the rounding-up rule to create significant distortions in reported trading volumes for an illiquid security regardless of its stock price.

Columns (3) and (4) summarize percent inflation rates for quintile 3 stock-days (closing price between \$13.575 and \$29.75) and quintile 4 stock-days (closing price between \$29.7504 and \$68.52). In both cases, there are roughly 1,200 stock-days having percent inflation of more than 1%, though the number of stock-days having extreme inflation is more modest than in column (2). As with column (2), however, the stock-days that do have percent inflation of more than 5% are associated with very few daily trades.

In column (5), we present percent inflation for quintile 5 stock-days (closing price greater than \$68.53). As exemplified by BRK.A, more expensive stocks should be at a higher risk of inflation due to the rounding-up rule, and under the 10% Scenario, column (5) reveals that 857 stock-days had percent inflation of 1% to 5% despite having an average of roughly 2,500 trades per day. However, as with the other columns, more extreme rates of percent inflation are limited to stock-days having far fewer daily trades.

Columns (6) through (10) present percent inflation results for the 90% Scenario. As expected, the overall level of inflation is less than in the 10% Scenario, though percent inflation in the range of 1% to 5% remains present across all price quintiles. More extreme rates of percent inflation are primarily confined to quintile 5 stock-days. As in the 10% Scenario, these extreme rates of inflation occur on stock-days having very few daily trades, raising the risk that the rounding-up rule can create severe distortions in reported trading volume.

6. Conclusions

Retail trading is changing and fractional shares are playing a growing role in this evolution. No longer the "nuisance" of times past, fractional shares are part of a new way of trading in which customers specify orders in dollars not shares, fintech apps sweep up spare change to invest in tiny amounts, and high share prices are no longer an impediment to retail stock ownership. Despite the growing importance of fractional share trades, our paper makes clear the challenges in understanding this new development—fractional share trades are largely opaque, with disparate reporting rules, exclusion from exchange trading, and "rounding-up" and "de minimis" rules all contributing to the difficulty in measuring or even observing these trades.

This paper developed a methodology for identifying fractional share trades in the consolidated market data. Our approach uses a latency-based digital footprint to estimate fractional share trades executed by Robinhood and Drivewealth, giving us a window into the trading of the largest fractional share broker dealers for retail investors. Our results show a surprising breadth to fractional share trading: high-priced stocks, meme stocks, IPOs, SPACs, and popular retail stocks now exhibit considerable numbers of these tiny trades. We also demonstrate, however, that fractional share trading is not evenly distributed across stocks, with many smaller, non-retail stocks less likely to feature such trading.

Nonetheless, we show that these tiny trades matter. Fractional share trades are predictive of future liquidity and volatility, suggesting an information content to these trades notwithstanding their small size. Equally intriguing, our results suggest that our measure of fractional share trading better captures this market information than do more standard measures of retail trading. We look forward to investigating why this occurs, and to examining the growth and dispersion of fractional share trading in future research.

What our research also highlights are the difficulties of ever knowing the full extent of fractional share trading given the current market data structure and the disparate rules on regulatory reporting. Fractional share trades do not fit into the current national market system of trade reporting, and efforts to make them do so with the "rounding-up" rule result in a distorted view of market trading volumes. As we have discussed, this distortion matters because, in addition to misleading investors, reported volume is utilized in a wide variety of legal contexts such as the rules relating to corporate stock repurchases and whether investors can bring a class action fraud lawsuit.

What then to do about this state of affairs? In the short run, a simple improvement would be to add a code to all trade reports indicating when a single share trade is a fractional share execution. While certainly not a complete solution, this reform would at least allow the market to adjust expectations of actual trading volume. Another short-term fix would be to lower the de minimis limits for the OTC Transparency data, thereby reducing (but, of course, not totally eliminating) the censored sampling of reported trades within those data. In the long run, greater change is needed. Without greater transparency into the fractional share trading process, research into this important market development will be severely hampered. Having two systems of clearing trades, one invisible and the other not is unsustainable. Similarly, excluding trade reports below one share from the tape only perpetuates the problem. Perhaps aggregating fractional trades until they actually reach one share and then putting the trade on the tape is a possible solution. Certainly, these tiny trades raise big issues for market debate.

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Appendix A

Classification Rules for Identifying Robinhood and Drivewealth Fractional Share Trades

We develop our classification rules for identifying Robinhood and Drivewealth fractional share trades in three steps. First, we strategically place fractional share trades in select issuers using the Robinhood and Drivewealth trading platforms, and we examine the resulting trade reports in the TAQ data to identify traces of a unique "signature" for Robinhood and Drivewealth fractional share trades. Second, with evidence of these "signatures," we then compare trading data for the period of time before Robinhood and Driveweath began reporting fractional share trades to the tape with trading data after such trading commenced to derive our exact classification rules. Finally, we use the OTC Transparency data to test the accuracy of each rule.

(1) Identifying Traces of the Robinhood and Drivewealth Signatures

We ground our classification rule in the simple observation that the trade reports for the fractional share trades of Robinhood and Drivewealth that appear in the TAQ data should have a number of distinctive characteristics. For one, because these trades are effectively internalized trades, they should be reported to one of three FINRA trade reporting facilities (TRFs). For another, due to the "rounding-up" rule, these trades should be for a single share. Lastly, there are reasons to believe that these trades will be characterized by unique trade reporting latencies, as we explain here.

Trade reports for U.S. equity securities listed on a national stock exchange are today required to be disseminated in real time to two securities information processors (SIPs) for public distribution. Additionally, since 2016 the trade reports redistributed by the SIPs (and recorded in the TAQ data) have included two timestamps: a participant timestamp (reflecting the time, generally in microseconds, that a trade or quote update occurred at the market center) and the "SIP" timestamp (reflecting the time, generally in microseconds, that a trade or quote was processed by the relevant SIP).

These two timestamps provide a means to estimate the time it takes for a trade report to travel from the executing broker to either the SIP located within NYSE's trading facility (for Tape A and Tape B securities) or the SIP located within Nasdaq's facility (for Tape C securities). Moreover, as shown in Bartlett & McCrary (2019), the transmission latencies across trading venues reveal distinct venue-specific latencies on account of the different geographical locations of each venue and the relevant SIP. For instance, a trade report relating to an execution in a Tape A security in Nasdaq's matching engine in Carteret, New Jersey will take longer to arrive at the NYSE's SIP in Mahwah, New Jersey, than a trade report relating to an execution in a Tape A security on the NYSE. Thus, to the extent a broker algorithmically executes fractional share trades at the same datacenter, it should, in principle, be possible to observe the broker's distinctive trade reporting latency. The large number of trades executed by Robinhood and Drivewealth should additionally increase the likelihood of identifying any such distinctive trade reporting latencies among all trades within the TAQ data.

We commence our search for evidence of distinct reporting latencies for Robinhood and Drivewealth by placing several trades for either $10.00 or $1.00 in a number of different securities using each firm's trading platform.[28] To facilitate locating each trade report in the TAQ data, most of our trades focused on purchasing fractional shares of BRK.A, which typically has fewer than 1,000 trades per day due to its unusually high stock price. However, we also placed fractional share trades in a variety of other stocks and ETFs (including AMZN, TSLA, and SIRI) to check for consistency in the two firm's reporting latencies across different securities.

In Table A1, we illustrate how five of these trades ultimately appeared in the TAQ data. The first two trades were placed with Robinhood and represented orders to buy $10 and $1 of BRK.A. As shown in columns (5) and (6), the trade confirmation for these orders indicated that they were filled at a per share price of $474,668.445 and $472,203.99, respectively. The trade confirmation also indicated that the first trade represented a fractional share purchase of 0.000021 of a share of BRK.A, while the second trade represented 0.000002 of BRK.A. Next, the TAQ trade file was examined to locate each trade based on its per share trade price and the minute at which it was filled according to the trade confirmation. As shown in columns (7) through (10), in each case a one share trade was located in the TAQ data that had the appropriate price and venue, nor did it have any special trade conditions which should be expected for fractional share trades.[29] Moreover, no other trades having these characteristics appears in the

[28] Drivewealth does not itself offer a brokerage platform to retail investors; instead, its brokerage platform is designed to provide securities trading functionality for other firms. We therefore use Cash App to execute trades through the Drivewealth platform.

[29] For instance, fractional share trades executed by retail investors will not sweep through the order book, nor will they be part of an integrated trading strategy (e.g., a VWAP trade). As such, aside from an indicator for an odd lot trade, fractional share trade reports should generally lack any other trade condition indicators. Other than for

TAQ trade data during the minute reflected on the trade confirmation. We therefore conclude that these two trades in TAQ reflect the buy orders we placed at Robinhood.

[Insert Table A1]

Columns (12) and (13) reflect the participant timestamp and SIP timestamp for these trades, which allows us to calculate the latency for each trade in Column (14). As shown there, the trades appear to reveal traces of distinct reporting latencies. Specifically, each trade report took roughly 210-250 milliseconds to travel from the execution venue to the relevant SIP.

Rows 3 and 4 provide similar examples for two trades placed with Cash App, which utilizes Drivewealth for executing trades. As shown in columns (3) and (4), each trade represented a buy order for $1.00 of each of NVR, Inc. and BRK.A (NVR, Inc. is similar to BRK.A insofar that relatively few trades occur per day due to its roughly $4,000 stock price, facilitating our ability to locate the trades in the TAQ data). As with the first two trades, we locate these trades in the TAQ data after searching for single share, FINRA trades that were executed at the correct minute and price, and we confirm that no other trades having these characteristics appear during that minute. As shown in Column (14), each trade likewise appears to have a distinctive reporting latency of roughly 25 milliseconds.

Using 2016 trading data, Bartlett & McCrary (2019) report that over 80% of FINRA-reported trades have a reporting latency of less than 20 milliseconds. Thus, the unusually large reporting latencies for these four trades provides an additional reason to believe that the fractional share trades executed by Robinhood and Drivewealth should be distinguishable from other trades on the basis of their reporting latencies. Moreover, as shown in column (11), examination of the trade reports for our sample of trades also revealed that Robinhood and Drivewealth appeared to differ in their choice of FINRA trade execution facility: Whereas the Drivewealth trades were reported to the FINRA trade reporting facility operated by NYSE, Robinhood's trades were each reported to the facility operated by Nasdaq.

Lastly, in rows 5A and 5B we present the results for a $10 buy order placed with Robinhood for Sirius XM Holdings Inc. (SIRI). At the time of the trade, the stock price for SIRI was less than $10 per share, allowing us to examine how dollar-based trades might yield a

BRK.A, the only trade condition we would expect to see would therefore be "I", indicating an odd lot trade. The size of a round lot for BRK.A is 1 share; therefore, there are no odd lot trades for this security.

fractional share execution. As shown in the table, Robinhood split the trade into a fractional component of 0.557109 shares as well as a whole share component. After locating the fractional component in the TAQ data, we calculated its reporting latency to be 227 milliseconds, similar to the first two Robinhood trades. In contrast, the whole share trade had a reporting latency of just 1.9 milliseconds and was sent to the NYSE TRF, rather than the Nasdaq TRF as had occurred with the fractional share trades executed by Robinhood. We therefore conclude that, consistent with Robinhood's trade disclosures, the whole share component of this trade was routed to a retail market maker for execution.

(2) Estimating the Distribution of Reporting Latencies.

While the foregoing evidence is suggestive that fractional share trades executed by Robinhood and Drivewealth possess distinctive, observable characteristics, the small, non-random sample of our trades naturally raises the question of whether these reporting latencies appear in other trades. Nor does this small sample of trades permit estimating the full distribution of reporting latencies that should be expected of Robinhood's and Drivewealth's fractional share trades.

To address both issues, we compare the reporting latency for all single share, non-exchange trades reported in TAQ during the final calendar quarter of 2020 with the same type of trades reported in TAQ during the final calendar quarter of 2021. In particular, trades during the last quarter of 2020 were made in a period prior to the time that Drivewealth commenced fractional share trades. Likewise, as discussed previously, Robinhood has disclosed in its securities filings that it did not commence reporting fractional share trades to FINRA until early 2021. Thus, comparing the distribution of reporting latencies for all trades during these two time periods—particularly in the region of 25 milliseconds and 200 milliseconds—can provide insight into whether the latencies observed for our sample of trades became more prevalent after Robinhood and Drivewealth began reporting fractional share trades to the tape. The fact that the reporting latencies for our trades were so large also points to a testable prediction: Trades with these observed latencies should be very rare in the 2020 Q4 sample but very common in the 2021 Q4 sample.

In Figure A1, we compare the distribution of reporting latencies for all single share, non-exchange trades reported for both time periods. Panel A presents trades reported to the Nasdaq TRF (Panel A), and Panel B presents trades reported to the NYSE TRF. Based on our individual stock trades, we surmised that the Nasdaq TRF receives trade reports from Robinhood; therefore, in Panel A, our interest is in the change in the frequency of trades with latencies of approximately 200 milliseconds between 2020 Q4 and 2021 Q4. In contrast, our preliminary analysis indicated that the NYSE TRF receives Drivewealth's trades; therefore, in Panel B, we focus on the distribution of latencies in the vicinity of 25 milliseconds.

As shown in Figure A1, Panel A is quite consistent with our prediction, and the sharp increase in single share, non-exchange trades in this area of the latency distribution during 2021 Q4 suggests a Robinhood latency "signature" of approximately 135 to 300 milliseconds. The results in Panel B are likewise consistent with the prediction, insofar that there is a sharp increase in trades with latencies between 20 milliseconds and 40 milliseconds during 2021 Q4, and (in contrast to Panel A) there also appears a general elevation of trades with latencies beyond 40 milliseconds. However, Panel B also reveals a sizeable number of trades with latencies between 25 and 30 during the 2020 Q4 period, suggesting that some trades reported to the NYSE TRF with latencies bearing the Drivewealth latency "signature" may reflect trade executions by other firms.

We conclude from this analysis that fractional share trades executed by Robinhood and Drivewealth should be discernable based on their reporting latencies; however, Panel A indicates that we may slightly undercount the presence of Robinhood's trades, while Panel B suggests we may slightly overcount Drivewealth's trades.

[Insert Figure A1]

Finally, examination of the full distribution of these data also reveals extensive positive skew in the 2021 Q4 distribution relative to the 2020 Q4, with the more recent data having a large number of trades with unusually large latencies. To illustrate, we define a trade report as "delayed" if it has a reporting latency of more than 1 second, and we plot the incidence of these delayed reports for our two periods in Figure A2. As shown in the figure, the incidence of such delayed trade reports spikes during the 2021 Q4 period for trade reports made to both the NYSE TRF and the Nasdaq TRF. Visual examination of the trade data indicate that the vast majority of these delayed trade reports are for trades executed at the commencement of the trading day. These delayed trade reports also have all of the other indicators of Robinhood or Drivewealth fractional share trades—in particular, with respect to trade condition and, in the case of

Robinhood, with respect to midpoint pricing (as discussed in Section 5 of the main paper). We therefore explore below whether the accuracy of our classification rules can be improved by including these delayed trades as fractional share executions by Robinhood and Drivewealth.

[Insert Figure A2]

We use these results to define separate classification rules for Drivewealth and Robinhood fractional share trade executions. For trades in the TAQ data, we classify a single share, non-exchange trade as a Drivewealth fractional share trade if the trade: (a) was reported to the NYSE TRF facility and (b) had a reporting latency of more than 20 milliseconds. In contrast, we classify a single share, non-exchange trade as a Robinhood fractional share trade if the trade (a) was reported to the Nasdaq TRF facility, and (b) had a reporting latency of between 135 and 300 milliseconds. For either Robinhood or Drivewealth trades, we also require that the trade condition be for an ordinary, non-ISO execution. In addition to these latency criteria, we also examine whether our classification accuracy for Robinhood can be enhanced by assigning all "delayed" trades executed at the opening of the trading day and reported to the Nasdaq TRF to be Robinhood fractional share trades.[30]

(3) Evaluating the Classification Rules

In conventional predictive modelling, classifiers such as the ones we constructed would be validated against a test dataset of trades that are known to have the target characteristic of interest (e.g., RH=0/1). Because we lack access to such a dataset, we turn instead to the OTC Transparency data to evaluate the accuracy of our classification rules. In particular, if the OTC Transparency data reveal that either Robinhood or Drivewealth executed only single share trades in a particular security during a trading week, the firm's weekly trades in that security likely reflect fractional share trade executions. We therefore evaluate the extent to which applying our classifiers to the TAQ trade data results in weekly trade estimates for Robinhood and Drivewealth for all such stock-weeks that match their disclosures for these stock-weeks in the OTC Transparency data.

To obtain our dataset of fractional share trade executions, we collect all stock-week disclosures in the OTC Transparency data for Robinhood and Drivewealth where the number of

[30] We do not explore this consideration for Drivewealth trades because we classify all single share, non-exchange trades as a Drivewealth fractional share trade if the reporting latency for a trade is greater than 20 milliseconds and the trade otherwise meets our classification criteria for Drivewealth fractional share trades.

reported shares traded for each firm for a stock-week is no greater than 100.01% the number of reported shares for the firm during that stock-week. Because neither Robinhood nor Drivewealth reported fractional share trades to FINRA prior to 2021, we focus on trades occurring from the week of Monday, January 4, 2021 to the week of Monday, March 28, 2022. These filters result in a dataset of 16,781 stock-weeks across 901 securities for Robinhood, and 17,593 stock-weeks across 1,037 securities for Drivewealth.[31]

In Figure A3, we use trades in BRK.A to illustrate the comparison we make given that BRK.A's unusually high stock price make it a popular stock for fractional share trading among users of Robinhood and Drivewealth. Indeed, during our sample period, these two firms are the only retail brokerage firms specifically listed as trading shares in BRK.A in the OTC Transparency data. Moreover, each firm consistently reports weekly trades and weekly shares traded in BRK.A that are the same number, indicating that these weekly trades represent fractional share executions. Nor should this fact be surprising given BRK.A's high stock price. In Panel A, we compare the weekly trades reported for Robinhood in the OTC Transparency data (represented by solid bars) with the weekly estimates for Robinhood's trades based on the trades in the TAQ data using the Robinhood classification rule described previously. Note that the rule we use for Panel A requires all trades to have a reporting latency of 135 to 300 milliseconds to be classified as a Robinhood trade; therefore, it does not classify any "delayed" trades as Robinhood trades.

Overall, Panel A indicates that this classification rule comes close to estimating the exact number of weekly trades in BRK.A reported for Robinhood in the OTC Transparency data, but only after the week of February 16, 2021. For instance, after March 1, the estimated number of Robinhood trades in Panel A is roughly 86% of the number reported for Robinhood in the OTC Transparency data. As suspected, however, the estimated number of Robinhood trades does appear to miss several trades each day, likely due to the fact that many trades executed at 9:30 a.m. have characteristics of Robinhood's fractional share trades (i.e., they are single share, non-exchange midpoint trades reported the Nasdaq TRF) but have very large reporting latencies, so they are not captured by this classification rule. In Panel B, we therefore present the results using

[31] Because Drivewealth continues to backfill its OTC disclosures, we expect these report numbers to change as it completes this process.

the same classification rule for Robinhood, but we allow a Robinhood trade to have either (a) a reporting latency of between 135 and 300 milliseconds, or (b) a reporting latency of greater than 1 second if the trade was executed between 8:55:00 and 09:30:10. As shown in Panel B, this modified rule results in a notable improvement in estimating the number of reported fractional share trades for Robinhood. Using this rule, the estimated number of Robinhood trades in Panel B is roughly 94% of the number reported in the OTC Transparency data from the week of March 1, 2021 through the week of March 28, 2022. In addition, the fact that both versions of our Robinhood classification rule shows a sudden increase in Robinhood trades over the two-week period beginning on February 16, 2021 also indicates that Robinhood commenced reporting its fractional share trades to the tape at around this time.

In Panel C, we present a similar comparison using the classification rule for Drivewealth. As with Panels A and B, the figure highlights a sudden increase in estimated Drivewealth trades beginning well into 2021, in the week of October 4, 2021. After this date, our classification rule for Drivewealth trades performs even better than the classifier for Robinhood trades. Across the twenty-five weeks between October 11, 2021 and March 28, 2022, the estimated number of weekly Drivewealth trades in BRK.A based on our classifier is identical to the number of trades reported for Drivewealth in the OTC Transparency data for twelve of these weeks. Moreover, on those weeks when the two numbers differed, the difference is typically less than 10 trades.

[Insert Figure A3]

Based on this analysis, we adopt a classification rule for Robinhood fractional share trades that includes "delayed" trades at the open. In main text, we provide a discussion of the performance of the Robinhood and Drviewealth classifiers using all stock-weeks in the OTC Transparency Data.

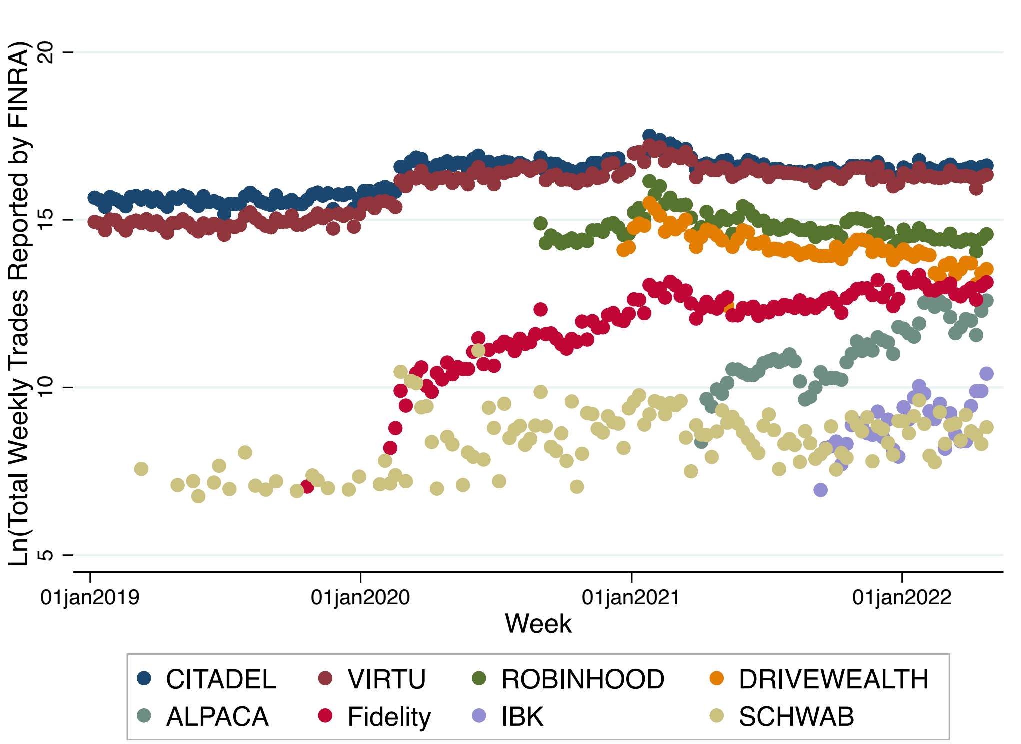
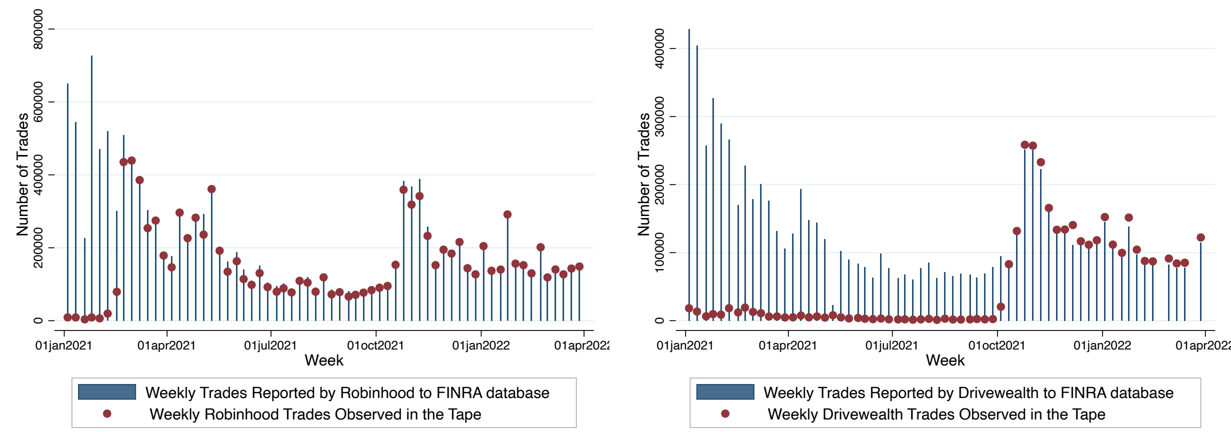
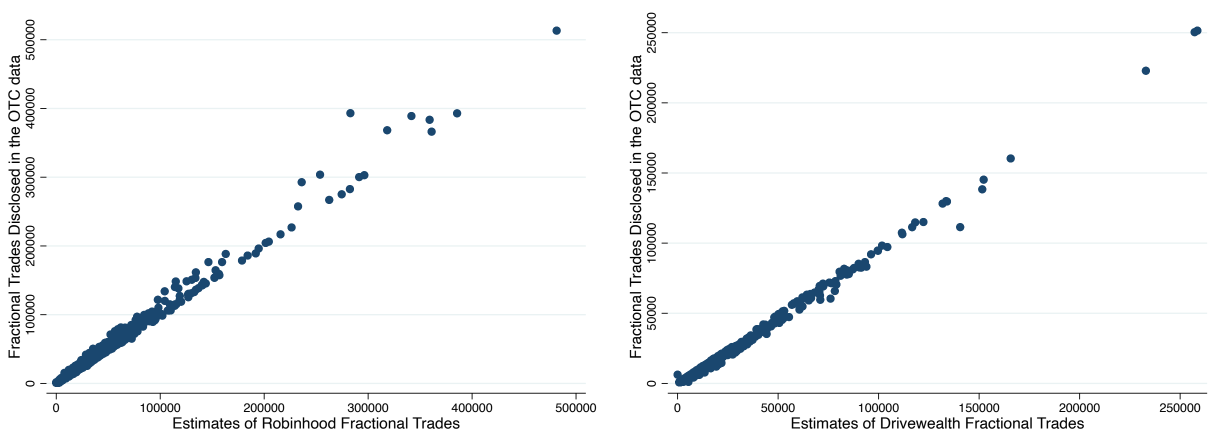


Figure 1: Weekly Trades Reported to FINRA by Brokerage Firm



A B

Figure 2 Fractional Share Trades in TSLA Reported in the OTC Transparency Data for Robinhood (Panel A) and Drivewealth (Panel B) vs. Fractional Share Trades Estimated by the RHDW Classifiers.



A B

Figure 3: Fractional Share Trades Reported in the OTC Transparency Data for Robinhood (Panel A) and Drivewealth (Panel B) vs. Fractional Share Trades Estimated in TAQ with the RHDW Classifiers.

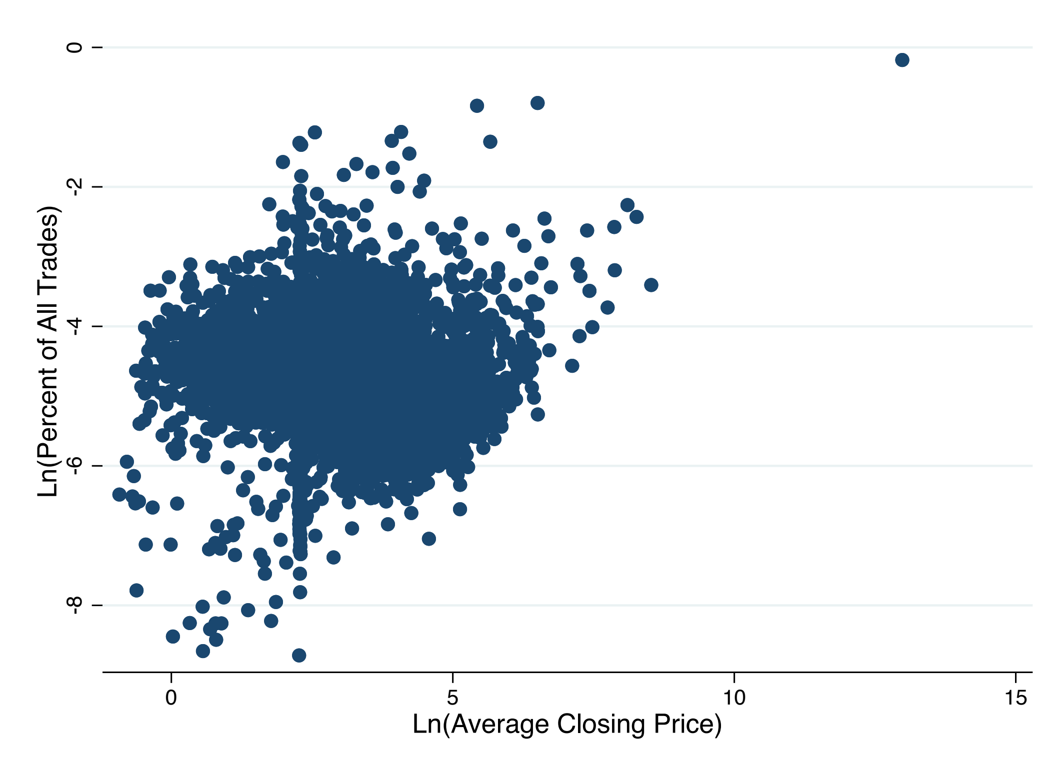


Figure 4: Fractional Trading and Stock Price

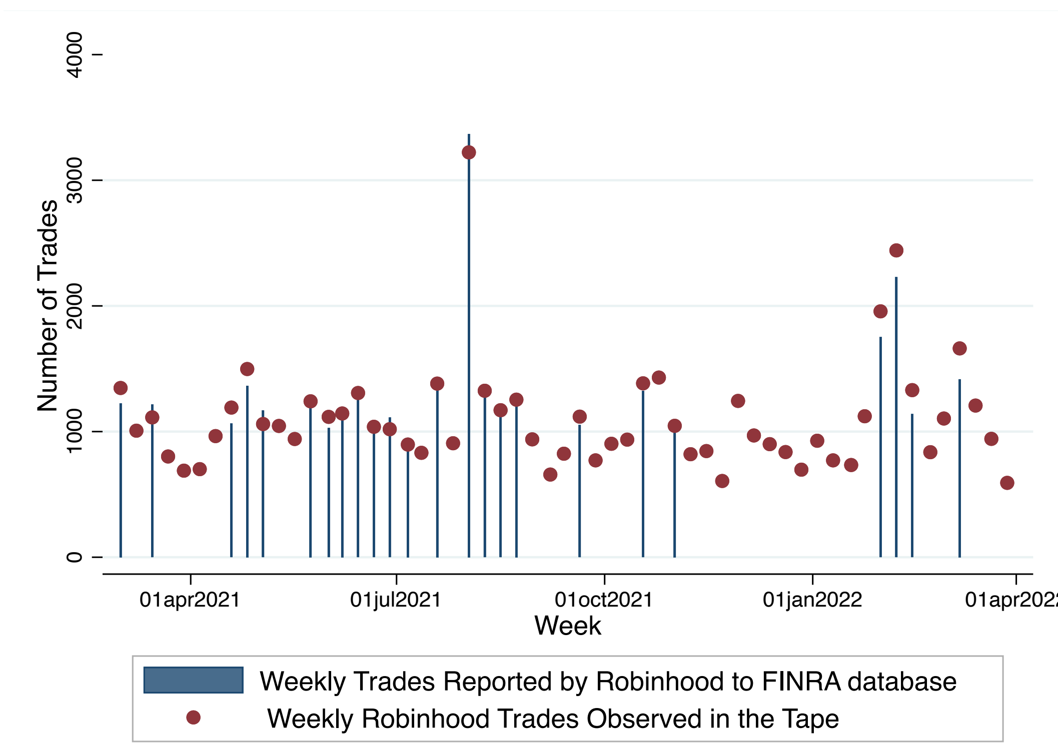
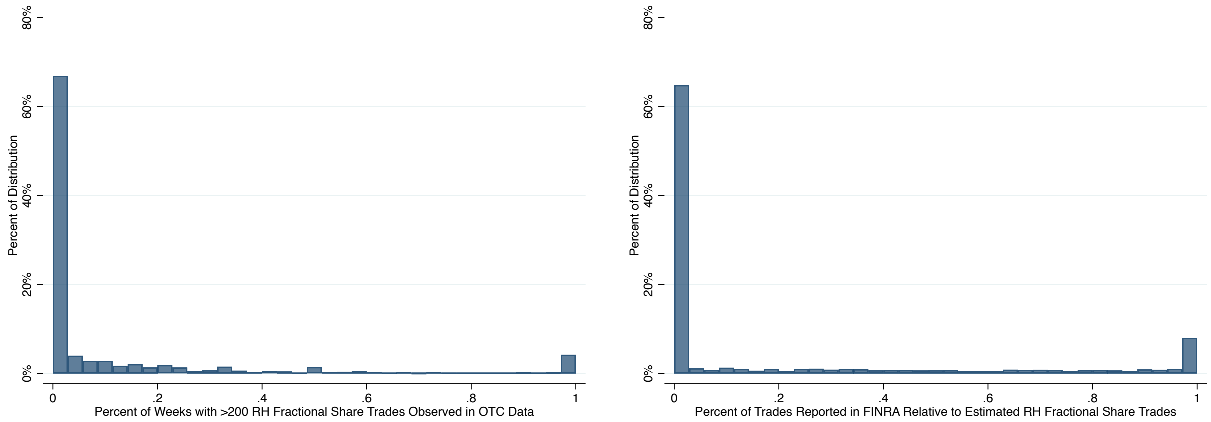


Figure 5: Weekly Robinhood Fractional Trades in CLX – OTC data vs. RH Classifier



A B

Figure 6: Percent of Robinhood Fractional Trades Captured by OTC Transparency Data. Panel A presents the percent of stock-weeks where the OTC Transparency data discloses Robinhood trades in sample stocks, relative to the number of stock-weeks where our classifer finds at least 250 RH fractional trades. Panel B presents the percent of Robinhood trades disclosed in the OTC Transparency data during the sample period for each sample stock, relative to the total number of Robinhood fractional trades for the stock based on the RH classifier.

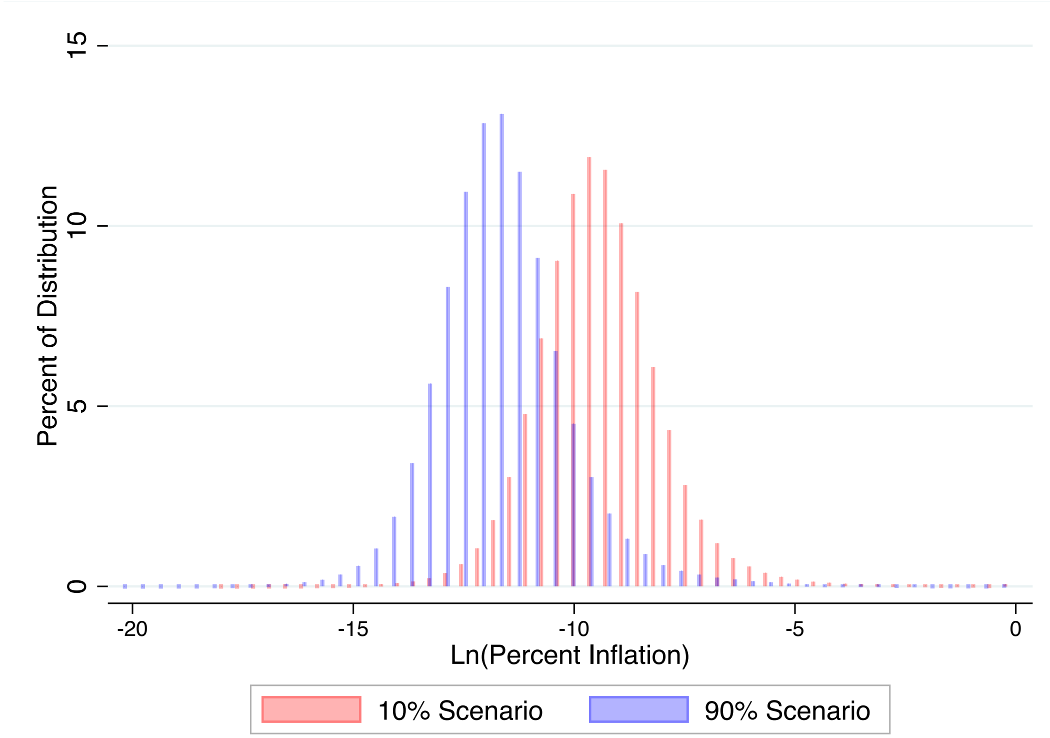
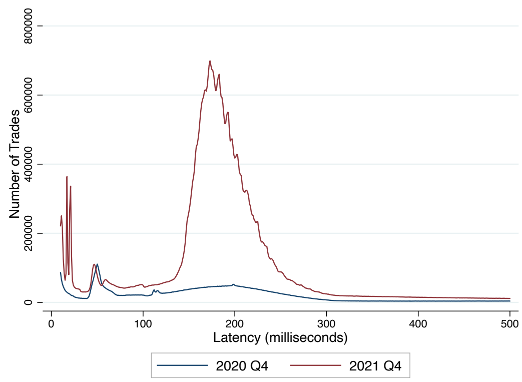
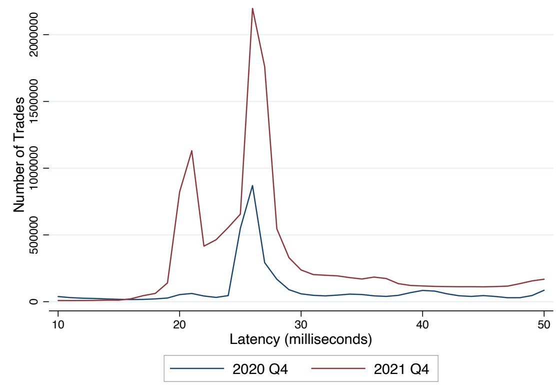


Figure 7: Distribution of Percent Inflation – 10% Scenario vs. 90% Scenario. Figure presents the distributions of the natural log of the average daily percent inflation for sample stock-days due to the rounding-up rule. Estimates for the 10% Scenario are presented in red; estimates for the 90% are presented in blue.



A



B

Figure A1: Reporting Latencies for Trades Reported to the Nasdaq TRF (Panel A) and the NYSE TRF (Panel B)

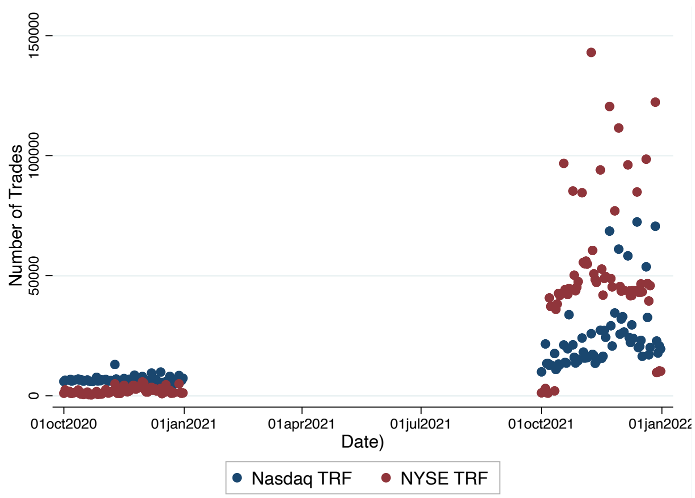
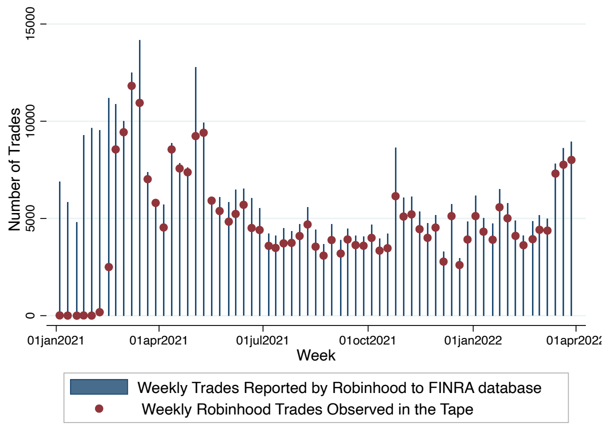
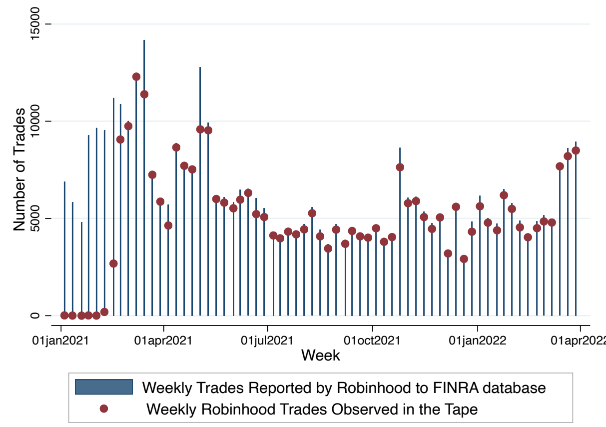


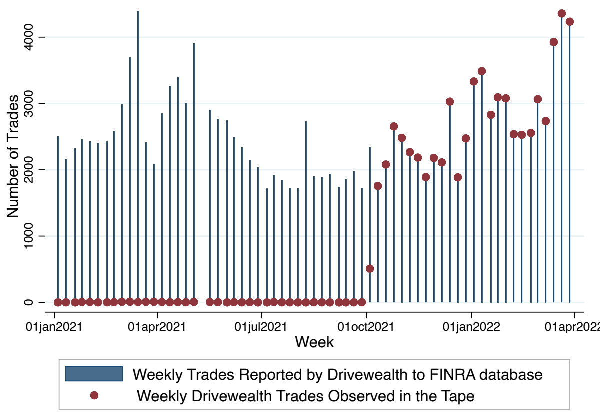
Figure A2: Incidence of Extended Reporting Delays, 2020 Q4 vs. 2021 Q4



A



B



C

Figure A3: Fractional Trades in BRK.A Reported in the OTC Transparency Data for Robinhood (Panels A & B) and Drivewealth (Panel C) vs. Fractional Trades Estimated in TAQ with Classification Rules

Table 1

This table reports the total trades and shares disclosed in the OTC Transparency Data between January 2019 and March 31, 2022 for the following eight brokers.

Reporting Firm	Trades (millions)	Shares (millions)	Average Trade Size
CITADEL SECURITIES LLC	2,266.31	759,419.10	335.09
VIRTU AMERICAS LLC	1,726.71	566,778.40	328.24
ROBINHOOD SECURITIES, LLC	245.11	257.05	1.05
DRIVEWEALTH, LLC	112.68	112.68	1.00
NATIONAL FINANCIAL SERVICES LLC	25.41	27.72	1.09
ALPACA SECURITIES LLC	4.21	4.27	1.01
CHARLES SCHWAB & CO., INC.	0.88	1.48	1.68
INTERACTIVE BROKERS LLC	0.15	0.15	1.00

Table 2

This table evaluates the accuracy of the classification rules for identifying fractional share trades executed by Robinhood and Drivewealth. Column 1 presents estimates from a regression of the total number of weekly trades reported for Robinhood in the OTC Transparency data on the total number of fractional share trades estimated to be executed by Robinhood using the TAQ trade data and the Robinhood classification rule. Column 2 presents estimates of the same regression applied to Drivewealth disclosures in the OTC Transparency data and using the Drivewealth classification rule. Sample period for Column 1 are the weeks commencing on March 1, 2021 through March 28, 2022, and the sample period for Column 2 are the weeks commencing on November 1, 2021 through March 28, 2022. Robust standard errors are in brackets.

	(1)	(2)
	Robinhood Disclosed Weekly Trades	Drivewealth Disclosed Weekly Trades
Weekly Estimates From TAQ	1.068***	0.926***
	[0.00996]	[0.00618]
Constant	-150.2***	-367.0***
	[49.05]	[28.48]
Observations	14,262	4,913
R-squared	0.991	0.995

Table 3

This table provides summary statistics of the incidence of fractional share trades for the fifty stocks having the large number of estimated fractional share trades in the sample. Average Trade Price is the mean price of all trades observed during the sample period. Market Capitalization is the mean daily value of the aggregate market value of equity for the stock. Average Daily Dollar Volume of Trades is the mean dollar value of trades in the stock during the sample period. Estimated Total of Fractional Share Trades is the total number of trades in the TAQ trade file during the sample period that are classified as fractional from the Robinhood and Drivewealth classification rules. % of All Trades that Are Fractional is the percentage of all trades in the stock during the sample period that are classified as fractional share executions using the Robinhood and Drivewealth classification rules. % of All Finra Trades that Are Fractional, % of All Single Share Trades that Are Fractional, and % of All Single Share Finra Trades that are Fractional show how the estimated number of fractional share trades compares to the total number of Finra trades, the total number of all single share trades, and the total number of all single share trades executed in a non-exchange venue, respectively.

Symbol	Company	(1) Average Trade Price	(2) Market Capitalization (millions$)	(3) Average Daily Dollar Volume of Trades	(4) Estimated Total of Fractional Share Trades	(5) % of All Trades that Are Fractional	(6) % of All Finra Trades that Are Fractional	(7) % of All Single Share Trades that Are Fractional	(8) % of All Single Share Finra Trades that Are Fractional
TSLA	TESLA INC	$810.48	$806,442	$21,606,400,000	13,200,000	6.7%	18.1%	26.4%	47.8%
AMC	AMC ENTERTAINMENT HOLDINGS INC	$25.87	$12,641	$2,830,454,883	8,884,018	5.4%	11.2%	39.6%	48.6%
AAPL	APPLE INC	$146.35	$2,418,453	$13,265,400,000	6,315,180	3.4%	9.7%	26.9%	41.3%
AMZN	AMAZON COM INC	$3,293.80	$1,666,783	$11,327,400,000	5,445,648	10.4%	25.3%	26.1%	45.9%
GME	GAMESTOP CORP NEW	$159.55	$11,769	$1,209,346,779	3,779,392	8.0%	17.1%	29.0%	41.0%
MSFT	MICROSOFT CORP	$280.86	$2,111,414	$8,133,720,852	3,332,237	3.3%	11.9%	22.5%	41.1%
NVDA	NVIDIA CORP	$403.77	$514,806	$9,005,162,863	2,859,166	2.4%	7.3%	16.1%	35.4%
FB	META PLATFORMS INC	$307.10	$729,701	$6,691,732,066	2,750,219	3.2%	10.6%	21.8%	41.2%
DIS	DISNEY WALT CO	$169.70	$308,271	$1,738,459,050	2,148,914	5.3%	14.2%	26.7%	43.0%
GOOG.L	ALPHABET INC	$2,530.32	$761,201	$4,088,884,424	2,032,333	7.6%	22.9%	18.1%	41.3%
COIN	COINBASE GLOBAL INC	$247.98	$37,938	$1,475,295,004	1,974,123	6.4%	15.5%	27.1%	43.0%
NFLX	NETFLIX INC	$530.20	$234,922	$2,344,896,944	1,901,672	5.8%	18.8%	25.1%	49.0%
F	FORD MOTOR CO DEL	$15.17	$59,481	$1,378,548,646	1,848,789	3.2%	7.2%	34.8%	44.2%
LCID	LUCID GROUP INC	$30.72	$50,419	$1,651,483,019	1,496,389	3.0%	7.9%	29.3%	48.4%
AMD	ADVANCED MICRO DEVICES INC	$104.75	$131,792	$7,067,705,078	1,347,389	1.1%	3.2%	16.0%	30.8%
PYPL	PAYPAL HOLDINGS INC	$230.26	$270,168	$2,366,720,635	1,313,531	2.5%	8.3%	17.5%	34.8%
RBLX	ROBLOX CORP	$79.51	$41,092	$1,232,293,008	1,280,883	3.2%	9.0%	24.1%	41.8%
MRNA	MODERNA INC	$232.01	$93,517	$3,288,110,634	1,278,924	2.6%	7.8%	19.8%	38.6%
PFE	PFIZER INC	$44.07	$246,736	$1,527,684,210	1,157,676	2.4%	7.2%	28.3%	41.9%
SQ	BLOCK INC	$212.41	$84,606	$2,145,417,978	1,125,622	2.7%	7.9%	19.5%	35.8%
HOOD	ROBINHOOD MARKETS INC	$28.34	$20,592	$514,864,839	1,096,700	4.9%	11.1%	38.8%	49.4%
PLTR	PALANTIR TECHNOLOGIES INC	$22.07	$38,675	$1,073,294,083	1,088,839	1.7%	4.3%	23.5%	35.8%
SNAP	SNAP INC	$56.54	$73,492	$1,251,146,167	1,064,501	2.5%	8.3%	29.6%	49.2%
RIVN	RIVIAN AUTOMOTIVE INC	$80.82	$71,232	$1,971,504,115	1,060,765	4.3%	10.5%	28.0%	45.1%
SPCE	VIRGIN GALACTIC HOLDINGS INC	$24.59	$5,984	$573,984,451	972,415	2.5%	6.7%	27.6%	43.0%
PLUG	PLUG POWER INC	$33.05	$18,180	$763,822,941	971,871	2.1%	6.9%	26.9%	46.2%
GE	GENERAL ELECTRIC CO	$59.49	$110,688	$808,598,609	926,350	2.7%	8.6%	28.2%	45.3%
WMT	WALMART INC	$141.49	$396,343	$1,162,821,486	910,953	3.7%	11.7%	26.9%	44.3%
KO	COCA COLA CO	$55.41	$239,084	$912,532,963	904,119	3.3%	10.5%	26.6%	42.2%
T	A T & T INC	$27.28	$194,718	$1,221,932,243	878,986	2.1%	5.3%	22.5%	33.4%
SBUX	STARBUCKS CORP	$108.56	$127,356	$747,183,823	863,031	3.6%	12.6%	20.0%	41.7%
BAC	BANK OF AMERICA CORP	$41.33	$348,242	$2,095,267,784	845,230	1.6%	5.8%	27.0%	43.5%
RIOT	RIOT BLOCKCHAIN INC	$31.23	$2,992	$479,010,055	793,539	2.8%	8.0%	29.7%	47.7%
GOOG	ALPHABET INC	$2,551.77	$818,045	$3,370,873,005	792,682	4.1%	13.6%	10.7%	27.5%
NKE	NIKE INC	$148.61	$189,413	$984,911,736	757,882	3.2%	12.0%	19.2%	43.1%
DKNG	DRAFTKINGS INC	$45.24	$18,184	$719,239,138	744,157	2.1%	6.2%	24.0%	40.5%
ABNB	AIRBNB INC	$165.95	$44,241	$1,045,837,759	743,014	2.7%	8.4%	16.7%	36.2%
TLRY	TILRAY BRANDS INC	$14.16	$4,552	$314,699,293	738,033	2.6%	6.7%	33.9%	45.4%
WISH	CONTEXTLOGIC INC	$9.72	$4,994	$279,661,122	707,051	2.5%	5.8%	37.6%	49.9%
OCGN	OCUGEN INC	$6.71	$1,315	$289,943,135	704,721	2.2%	5.3%	36.1%	50.0%
CLOV	CLOVER HEALTH INVESTMENTS CORP	$7.63	$1,507	$289,851,669	698,430	2.4%	5.7%	35.3%	48.8%
V	VISA INC	$221.01	$372,244	$1,910,772,444	678,534	2.1%	7.9%	14.8%	34.4%
DWAC	DIGITAL WORLD ACQUISITION CORP	$59.61	$1,790	$664,620,070	662,796	4.1%	9.9%	28.4%	43.3%
FCEL	FUELCELL ENERGY INC	$9.42	$3,205	$220,801,058	630,049	2.3%	6.6%	33.9%	51.0%
SOFI	SOFI TECHNOLOGIES INC	$15.87	$12,793	$524,460,302	627,499	2.0%	5.1%	26.6%	40.5%
UPST	UPSTART HOLDINGS INC	$153.45	$12,168	$971,915,008	622,307	2.6%	7.1%	18.1%	35.9%
CHPT	CHARGEPOINT HOLDINGS INC	$22.14	$6,385	$181,922,027	596,274	3.3%	8.2%	29.0%	43.3%
TWTR	TWITTER INC	$55.36	$44,190	$869,344,260	588,826	1.7%	6.9%	20.1%	42.0%
BA	BOEING CO	$219.84	$128,225	$2,589,437,328	584,008	1.5%	4.6%	12.6%	27.1%
CCIV	CHURCHILL CAPITAL CORP IV	$24.36	$5,042	$415,924,536	581,494	4.4%	8.6%	32.7%	45.0%
Total for top 50:					89,307,161	3.6%	10.1%	25.1%	42.8%
Total for all other stocks:					103,797,228	1.0%	3.8%	9.3%	29.6%
Total :					193,104,389	1.5%	5.3%	13.2%	34.5%

Table 4

This table summarizes the distribution of fractional share trades and the distribution of retail trades (calculated using the BJZZ metric) across sample stocks. For each stock in the sample, the total number of RHDW fractional share trades was estimated between March 1, 2021 and March 31, 2022 using the RH and DW classification rules, and stocks were ranked from highest to lowest by the total RHDW fractional shares for each stock. A similar ranking was conducted based on the total retail trades observed for each stock during this time period using the BJZZ metric. Top 50 refers to the 50 stocks that had the highest total number of trades for each ranking. Each subsequent row expands the sample by including additional stocks according to their position in the ranking. Percent of All Fractional Share Trades is the total number of RHDW fractional share trades represented by stocks in the group relative to all observed RHDW fractional share trades. Percent of All Retail Trades is the total number of retail trades (measured using the BJZZ metric) represented by stocks in the group relative to all observed retail trades.

	(1)	(2)	(3)
Group of Stocks	Percent of All Fractional Share Trades	Percent of All Retail Trades	Difference
Top 50	46.54%	33.05%	13.50%
Top 100	57.26%	43.98%	13.28%
Top 200	67.43%	55.65%	11.78%
Top 500	79.23%	71.52%	7.70%
Top 1000	87.17%	84.04%	3.12%
Top 2000	94.50%	94.73%	-0.23%
Top 3000	98.33%	98.71%	-0.39%

Table 5

This table reports regression results for a model of daily RHDW fractional share trading as a function of stock characteristics. The sample consists of the common stocks of all U.S. firms contained in CRSP, and the sample period is March 1, 2021 through March 31, 2022. All variables are measured for stock i on day t. The outcome in all models is the natural log of the daily number of RHDW fractional share trades. *Ln(retail flow)* is the natural log of the estimated number of daily retail trades based on the BJZZ metric. *Ln(price)* is the natural log of the daily closing price. *Ln(mktcap)* is the natural log of market capitalization. *New* is an indicator for whether the stock has traded for fewer than 180 calendar days. *Ln(age)* is the natural log of the number of days the stock has traded. *Ln(volatility)* is the natural log of the stock's return volatility over the past thirty trading days. *Ln(turnover)* is the natural log of the volume of shares traded relative to the number of outstanding shares. *Ln(short interest)* is the natural log of the most recent bi-monthly short-interest ratio. In columns (1) and (2), indicators are also included for stock exchange listing. Robust standard errors are included in brackets. *** p<0.01, ** p<0.05, * p<0.1

	(1)	(2)	(3)
Ln(retail flow)	0.639***	0.624***	0.481***
	[0.00141]	[0.00142]	[0.00456]
Ln(price)	0.168***	0.172***	0.192***
	[0.00150]	[0.00148]	[0.0344]
Ln(mktcap)	0.125***	0.134***	0.328***
	[0.00149]	[0.00149]	[0.0305]
New	0.0571***	0.0984***	
	[0.00507]	[0.00488]	
Ln(age)	-0.0639***	-0.0556***	0.283***
	[0.000783]	[0.000761]	[0.0163]
Ln(volatility)	0.123***	0.133***	0.0831***
	[0.00139]	[0.00141]	[0.00376]
Ln(turnover)	0.0441***	0.0818***	0.159***
	[0.00163]	[0.00166]	[0.00632]
Ln(short interest)	0.0320***	0.0265***	-0.0189***
	[0.000988]	[0.000984]	[0.00661]
Ex=NYSE MKT	-0.0197***	-0.0057	
	[0.00643]	[0.00646]	
Ex=Nasdaq	0.124***	0.126***	
	[0.00214]	[0.00206]	
Constant	-1.953***	-2.072***	-8.346***
	[0.0215]	[0.0213]	[0.553]
Day Fixed Effects:	N	Y	Y
Stock Fixed Effects:	N	N	Y
Observations	1,046,706	1,046,706	1,046,706
R-squared	0.69	0.707	0.231
Number of stocks			4,540

Robust standard errors in brackets
*** p<0.01, ** p<0.05, * p<0.1

Table 6

This table summarizes several trade execution measures for fractional share executions by Robinhood and Drivewealth in sample stocks between November 1, 2021 through March 31, 2022. Trade direction was based on the trade's price relative to the midpoint of the NBBO based on the NBBO published by the SIPs at the time of the trade based on the participant timestamp for the trade. Trades were classified as *Buy At NBO* or *Sell At NBB* based on whether the price of the trade was equal to the NBO or NBO published by the SIPs at the time of the trade (also based on the participant timestamp for the trade.). Trades were excluded from the table if the NBBO was locked or crossed at the time of the trade.

	Robinhood Fractoinal Share Trades					
	Fractional Share Trades	Buy	Sell	Midpoint	Buy At NBO	Sell At NBB
9:30 AM to 10:00 AM	10,279,952	18.20%	26.42%	55.25%	3.05%	4.03%
10:00 AM through Close of Regular Session	38,365,722	13.13%	12.26%	74.55%	6.69%	5.91%
Early Morning Session	772,212	2.04%	5.97%	91.91%	0.23%	2.46%
After Market Session	1,616,174	2.63%	2.65%	94.58%	0.19%	0.22%

	Drivewealth Fractoinal Share Trades					
	Fractional Share Trades	Buy	Sell	Midpoint	Buy At NBO	Sell At NBB
9:30 AM to 10:00 AM	13,140,556	63.59%	35.32%	1.07%	39.26%	19.75%
10:00 AM through Close of Regular Session	21,312,864	63.39%	35.82%	0.75%	46.63%	27.10%
Early Morning Session	74,751	48.08%	25.14%	2.37%	26.46%	13.89%
After Market Session	634	17.19%	81.86%	0.63%	11.51%	19.72%

Table 7

This table reports horserace regressions comparing the extent to which the liquidity and volatility for stock i on day t+1 can be predicted from (a) the total volume of retail trading in stock i on day t estimated using the BJZZ metric, and (b) the total number of RHDW trades in stock i on day t. In columns (1) through (4), the outcome variable is the natural log of average percent effective spread for stock i on day t+1. In columns (5) through (8), the outcome variable is the natural log of the intraday volatility of trades for stock i on day t+1. In columns (9) through (12) the outcome variable is the natural log of the implied volatility for call options on stock i on day t+1. The sample period for columns (1) through (8) is March 1, 2021 through March 31, 2022. The sample period for columns (9) through (12) is March 1, 2021 through December 31, 2021. The variables of interest are: Retail, which is the natural log of the dollar volume of retail trades for stock i on day t estimated using BJZZ; Fractional, which is the natural log of the total number of RHDW fractional trades for stock i on day t; and the interaction of these variables with *Top 100*, an indicator for whether stock *i* ranks as among the 100 stocks with the most RHDW fractional share trades between March 1, 2021 and March 31, 2022. All regressions include day and stock fixed effects as well as the following time varying controls measured at day *t-1*: Volume (the natural log of the dollar volume of trades), Trades (the natural log of the total number of trades), Prior Day Return (previous day's return), Prior Week Return (previous week's return), Prior Month Return (previous month's return), Mktcap (the natural log of the stock's aggregate equity value), Turnover (the natural log of the previous month's trading volume scaled by its outstanding shares), Prior Month Volatility (the previous month's daily return volatility), and BTM (the book-to-market ratio). We additionally include the lagged dependent variable in all regressions. The standard errors (in parentheses) are double clustered at day and stock level. *** p<0.01, ** p<0.05, * p<0.1

VARIABLES	(1) Effective Spreads	(2) Effective Spreads	(3) Effective Spreads	(4) Effective Spreads	(5) Intraday Volatility	(6) Intraday Volatility	(7) Intraday Volatility	(8) Intraday Volatility	(9) Implied Volatility	(10) Implied Volatility	(11) Implied Volatility	(12) Implied Volatility
Retail	0.000974		0.000276	0.000348	0.00872***		0.00743***	0.00776***	0.00909***		0.00853***	0.00836***
	(0.000836)		(0.000834)	(0.000836)	(0.00193)		(0.00193)	(0.00193)	(0.00119)		(0.00119)	(0.00119)
Fractional		0.00770***	0.00768***	0.00733***		0.0157***	0.0152***	0.0140***		0.00402***	0.00331***	0.00300***
		(0.000815)	(0.000812)	(0.000806)		(0.00201)	(0.00201)	(0.00198)		(0.000867)	(0.000863)	(0.000860)
Retail Volume X Top100				-0.0255***				-0.0975***				0.00937*
				(0.00787)				(0.0229)				(0.00556)
Fractional X Top100				0.0667***				0.257***				0.0306***
				(0.0119)				(0.0360)				(0.00688)
Volume	-0.0312***	-0.0282***	-0.0284***	-0.0283***	-0.0630***	-0.0517***	-0.0578***	-0.0574***	-0.0324***	-0.0251***	-0.0310***	-0.0308***
	(0.00296)	(0.00285)	(0.00301)	(0.00302)	(0.00767)	(0.00758)	(0.00788)	(0.00792)	(0.00405)	(0.00389)	(0.00413)	(0.00416)
Trades	0.00353	-0.00373	-0.00378	-0.00451	0.00878	-0.00472	-0.00605	-0.00917	0.0522***	0.0514***	0.0490***	0.0476***
	(0.00429)	(0.00447)	(0.00446)	(0.00449)	(0.0119)	(0.0125)	(0.0125)	(0.0126)	(0.00506)	(0.00537)	(0.00528)	(0.00535)
Prior Day Return	0.162***	0.165***	0.165***	0.167***	0.323***	0.330***	0.328***	0.336***	0.0470***	0.0509***	0.0485***	0.0479***
	(0.0158)	(0.0158)	(0.0158)	(0.0157)	(0.0384)	(0.0384)	(0.0384)	(0.0380)	(0.0132)	(0.0133)	(0.0132)	(0.0131)
Prior Week Return	-0.0465***	-0.0462***	-0.0462***	-0.0460***	-0.0941***	-0.0935***	-0.0937***	-0.0928***	-0.00913	-0.00870	-0.00900	-0.00964
	(0.00735)	(0.00732)	(0.00732)	(0.00730)	(0.0182)	(0.0181)	(0.0181)	(0.0180)	(0.00644)	(0.00642)	(0.00642)	(0.00638)
Prior Month Return	0.0204***	0.0198***	0.0197***	0.0200***	0.0649***	0.0642***	0.0636***	0.0645***	-0.00390	-0.00314	-0.00414	-0.00481
	(0.00445)	(0.00444)	(0.00444)	(0.00441)	(0.0110)	(0.0110)	(0.0110)	(0.0108)	(0.00434)	(0.00433)	(0.00433)	(0.00430)
Mktcap	-0.0964***	-0.0987***	-0.0987***	-0.0993***	-0.221***	-0.227***	-0.226***	-0.229***	-0.0121	-0.0135	-0.0128	-0.0143
	(0.00813)	(0.00817)	(0.00817)	(0.00818)	(0.0207)	(0.0209)	(0.0209)	(0.0210)	(0.0115)	(0.0115)	(0.0115)	(0.0115)
Turnover	-0.0607***	-0.0619***	-0.0619***	-0.0621***	-0.160***	-0.162***	-0.163***	-0.164***	0.0143***	0.0140***	0.0137***	0.0135***
	(0.00184)	(0.00184)	(0.00183)	(0.00183)	(0.00447)	(0.00445)	(0.00444)	(0.00445)	(0.00252)	(0.00247)	(0.00247)	(0.00248)
Prior Month Volatility	0.454***	0.438***	0.438***	0.435***	1.087***	1.065***	1.059***	1.050***	0.363***	0.364***	0.355***	0.347***
	(0.0324)	(0.0315)	(0.0315)	(0.0316)	(0.0820)	(0.0810)	(0.0806)	(0.0809)	(0.0311)	(0.0314)	(0.0307)	(0.0310)
BTM	-0.0283***	-0.0275***	-0.0275***	-0.0278***	-0.0793***	-0.0781***	-0.0779***	-0.0793***	0.00633	0.00640	0.00665	0.00602
	(0.00713)	(0.00714)	(0.00714)	(0.00715)	(0.0187)	(0.0188)	(0.0188)	(0.0189)	(0.0100)	(0.0100)	(0.0100)	(0.0101)
Lagged DV	0.566***	0.566***	0.566***	0.565***	0.466***	0.465***	0.465***	0.462***	0.446***	0.446***	0.445***	0.444***
	(0.00633)	(0.00633)	(0.00632)	(0.00634)	(0.00701)	(0.00709)	(0.00707)	(0.00715)	(0.0146)	(0.0146)	(0.0146)	(0.0146)
Constant	-0.801***	-0.749***	-0.749***	-0.743***	-3.160***	-3.060***	-3.065***	-3.050***	-0.0271	-0.000103	-0.00731	0.0119
	(0.118)	(0.118)	(0.118)	(0.118)	(0.287)	(0.289)	(0.289)	(0.290)	(0.154)	(0.155)	(0.154)	(0.155)
Observations	972,624	972,624	972,624	972,624	972,523	972,523	972,523	972,523	508,725	508,725	508,725	508,725
R-squared	0.957	0.957	0.957	0.957	0.936	0.936	0.936	0.936	0.899	0.899	0.899	0.899

Table 8

This table provides estimate of the total dollar value of inflated trading volume between March 1, 2021 and March 31, 2022 for sample stocks due RHDW fractional share trades and the rounding-up rule. Listed are the top twenty stocks within the sample by dollar value of inflated volume. The *10% Scenario* assumes that the dollar value of each fractional share trade is equal to the lesser of $240 and 10% of the reported trade price. The *90% Scenario* assumes that the dollar value of each fractional share trade is equal to the lesser of $240 and 90% of the reported trade price.

Symbol	Company	Average Closing Price	Reported Dollar Volume ($billions)	Dollar Inflation (10% Scenario) ($ billions)	Dollar Inflation (90% Scenario). ($ billions)	Percent Inflation (10% Scenario)	Percent Inflation (90% Scenario)
BRK.A	BERKSHIRE HATHAWAY INC DEL	$435,228.00	$212	$172	$172	81.13%	81.13%
AMZN	AMAZON COM INC	$3,302.43	$3,130	$16.60	$16.60	0.53%	0.53%
TSLA	TESLA INC	$810.85	$5,950	$10.30	$8.23	0.17%	0.14%
GOOG.L	ALPHABET INC	$2,614.17	$1,120	$4.98	$4.97	0.44%	0.44%
GOOG	ALPHABET INC	$2,637.24	$925	$1.94	$1.94	0.21%	0.21%
NFLX	NETFLIX INC	$529.26	$646	$0.90	$0.54	0.14%	0.08%
NVDA	NVIDIA CORP	$382.74	$2,480	$0.89	$0.34	0.04%	0.01%
MSFT	MICROSOFT CORP	$287.87	$2,240	$0.88	$0.20	0.04%	0.01%
AAPL	APPLE INC	$148.37	$3,660	$0.86	$0.10	0.02%	0.00%
FB	META PLATFORMS INC	$312.82	$1,840	$0.73	$0.19	0.04%	0.01%
GME	GAMESTOP CORP NEW	$171.15	$333	$0.65	$0.08	0.20%	0.02%
COIN	COINBASE GLOBAL INC	$247.88	$356	$0.51	$0.12	0.14%	0.03%
CMG	CHIPOTLE MEXICAN GRILL INC	$1,613.84	$121	$0.42	$0.40	0.34%	0.33%
DIS	DISNEY WALT CO	$168.23	$906	$0.33	$0.04	0.04%	0.00%
MRNA	MODERNA INC	$243.63	$480	$0.33	$0.09	0.07%	0.02%
MELI	MERCADOLIBRE INC	$1,430.48	$198	$0.30	$0.27	0.15%	0.14%
AMC	AMC ENTERTAINMENT HOLDINGS INC	$28.71	$783	$0.28	$0.03	0.04%	0.00%
PYPL	PAYPAL HOLDINGS INC	$226.49	$652	$0.24	$0.03	0.04%	0.00%
BKNG	BOOKING HOLDINGS INC	$2,320.90	$233	$0.24	$0.24	0.10%	0.10%
COST	COSTCO WHOLESALE CORP NEW	$452.89	$281	$0.23	$0.12	0.08%	0.04%
		Total:	$26,546	$214	$207		

Table 9

This table summarizes the frequency of stock-days based on their percent inflation and their stock price under the 10% Scenario and the 90% Scenario. Data are limited to stock-days where the percent inflation for either the 10% Scenario or the 90% Scenario is greater than 1%. *Percent Inflation Range* is the percent inflation for a stock-day based on either the 10% Scenario or the 90% Scenario. *Price Quintiles are defined as 1: closing price<=$6.73; 2:$6.730<closing price<=$13.57; 3: $13.57<closing price<=$29.75; 4:$29.75<closing price<=$68.52; 5: closing price>$68.52.* Columns (1) through (5) present frequencies for the 10% Scenario while Columns (6) through (10) present frequencies for the 90% Scenario. The number in parentheses represents the average daily number of trades for the stock-days assigned to the Inflation-Range X Price Quintile bin.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	10% Scenario					90% Scenario				
	Price Quintile					Price Quintile				
Percent Inflation Range	1	2	3	4	5	1	2	3	4	5
1% - 5%	45	1111	948	806	867	1	588	151	220	630
	(235.76)	(54.31)	(87.3)	(188.02)	(2654.84)	(6)	(44.46)	(10.66)	(16.88)	(3400.46)
5% - 10%	2	266	158	155	309		224	30	41	245
	(133.5)	(48.03)	(24.69)	(31.42)	(169.91)		(96.73)	(4.97)	(7.68)	(199.72)
10% - 15%		158	59	57	116		125	4	11	76
		(58.1)	(14.25)	(15.02)	(76.67)		(1.38)	(1.5)	(3)	(101.8)
15% - 20%	1	112	28	49	73					28
	(6)	(21.94)	(9.5)	(23.2)	(19.07)					(28.75)
20% - 25%		85	16	33	63					32
		(39.56)	(5.94)	(20.36)	(10.11)					(10.84)
25% - 30%		56	11	17	49					24
		(15.64)	(9.36)	(9.41)	(11.98)					(12.25)
30% 35%		56	5	22	38					22
		(59.66)	(3.4)	(9.77)	(11.11)					(10.77)
35% - 40%		21	8	12	30					16
		(202.52)	(5.13)	(14.58)	(10.63)					(13.81)
40% - 45%		87	7	13	42					12
		(3)	(2.57)	(6.77)	(10.74)					(13.58)
45% - 50%		40	8	9	21					6
		(60.13)	(7.63)	(5.78)	(9.05)					(11.5)
50% - 55%		5	3	5	30					7
		(169.8)	(12.67)	(7)	(11.87)					(7.86)
55% - 60%		21	2	4	21					3
		(3.95)	(4.5)	(4.25)	(9.33)					(7)
60% - 65%		15	2	3	20		1			3
		(5.07)	(3)	(5.33)	(10.5)		(25)			(6.33)
65% - 70%		7		5	12				1	6
		(340.86)		(16.2)	(10.08)				(17)	(4.33)
70% - 75%		3	2	1	8					1
		(1140.33)	(8)	(17)	(5.75)					(5)
75% - 80%		1		3	3					1
		(1359)		(15.33)	(7.67)					(5)
80% - 85%		4		2	2					
		(442.5)		(23.5)	(10)					
85% - 90%		234	16	20	28					
		(40.46)	(1.44)	(1.95)	(2.57)					
Total	48	2282	1273	1216	1732	1	938	185	273	1112

Table A1

This table provides examples for five trades used to examine the trade characteristics of Robinhood and Drivewealth trades. The date, broker used, trade value, and stock traded are listed in columns (1) through (4). Columns (5) and (6) reports the fraction of a share purchased and the per share price of the trade based on the trade confirmation for each trade. Columns (7) through (14) provide data obtained upon locating the trade in the TAQ trade file. *EX* is the venue reporting the trade. *TR_SCOND* is the trade condition (if any) reported for the trade ("I" indicates an odd lot trade, which is a trade for fewer than 100 shares for all stocks other than BRK.A, which has a round lot equal to 1 share). *SIZE* is the size of the trade, and *PRICE_TAQ* is the price of the trade, each as reported in the TAQ trade data. *TR_RF* is the trade reporting facility to which the trade report was sent. *PART_TIME* and *TIME_M* are the participant timestamp and the SIP timestamp, respectively, and *Latency* is the number of milliseconds between the PART_TIME and TIME_M.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
			From Trade Confirmation:				From TAQ Trade File:							
Trade #	DATE	Broker	Trade Value	Symbol	Fraction Purchaed	Price	EX	TR_SCOND	SIZE	PRICE_TAQ	TR_RF	PART_TIME	TIME_M	Latency
1	20220203	Robinhood	$10.00	BRK.A	0.000021	$474,668.445	D		1	$474,668.445	Nasdaq	13:12:14.165777000	13:12:14.417748480	251.97
2	20220204	Robinhood	$1.00	BRK.A	0.000002	$472,203.990	D		1	$472,203.990	Nasdaq	10:11:50.522135000	10:11:50.732623616	210.49
3	20220303	CashApp	$1.00	NVR	0.000200	$4,987.730	D	I	1	$4,987.730	NYSE	13:06:18.222000000	13:06:18.248067840	26.07
4	20220303	CashApp	$1.00	BRK.A	0.000002	$489,804.680	D		1	$489,804.680	NYSE	12:40:07.742000000	12:40:07.765288448	23.29
5A	20220307	Robinhood	$10.00	SIRI	0.557109	$6.365	D	@ I	1	$6.365	Nasdaq	13:49:38.714177000	13:49:38.941501430	227.32
5B	20220307	Robinhood		SIRI	1.000000	$6.365	D	@ I	1	$6.365	NYSE	13:49:38.816441424	13:49:38.818406687	1.97