Principal Trading Procurement: Competition and Information Leakage*

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Abstract

We model procurement auctions held by institutional traders seeking fulfillment for large trades. The dealer who wins such an auction might fill the order out of inventory or access the market for additional volumes. How many dealers should the trader contact? There is a general tradeoff: an additional dealer intensifies competition and may improve matchmaking, but also intensifies information leakage. We show that information leakage can be an endogenous search friction in that the trader does not always contact all available dealers. There is also a question of information design: what should the trader reveal about her desired trade? In the model, it is optimal to provide *no* information at the bidding stage. There are also implications for market design and regulation.

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1 Introduction

Institutional traders often have relationships with multiple dealers. When seeking execution of large orders, they may solicit quotes from more than one, in search of the best quote. But each additional dealer contacted comes with a tradeoff. On the one hand, the additional dealer may reduce the ultimate cost of procurement by intensifying competition among the dealers for the trader's business. The additional dealer may also be able to provide fulfillment more efficiently—for instance the dealer might, by virtue of an existing position, be able to internalize the trader's order rather than expose it to the market. On the other hand, the very act of reaching out to an additional dealer creates information leakage, which can be costly because a losing dealer can leverage knowledge of the trader's presence to front-run on the market.

Motivated by this issue, we propose a model of these tradeoffs. A client wishes to either buy or sell a large position of a security and seeks fulfillment through a dealer. Our main results answer how this client optimally orchestrates her procurement process. How many dealers should she contact? We show that the client does not always find it optimal to contact all available dealers. In that sense, the aforementioned concern about information leakage (and the front-running it leads to) can be interpreted as an endogenous search friction. What information should she provide about her desired trade while soliciting quotes? Optimizing over all information structures, we show it is unambiguously optimal to provide no information—precisely to mitigate front-running. In addition to shedding light on this search problem, our results also have implications for the design and regulation of trading protocols employed in a variety of markets.

Model and equilibrium. In the model, the client issues a request for quotes (RFQ), contacting either one or two dealers. Each dealer is either long or short in the security, where these positions are unknown to the client (although, for simplicity, we assume that these positions are common knowledge among the dealers). The contacted dealers provide two-sided quotes. Using only the relevant side of the submitted quotes, the client conducts a sealed-bid, second-price auction (with reserve). The winning dealer then learns the client's desired trade, while the losing dealer can only make inferences based on the RFQ itself, together with the auction's outcome. The winning dealer might fulfill the client's order in either of two ways. First, he might internalize the order against his inventory. However, we assume position limits for the dealers, so that internalization is possible only if the dealer is long (short) and the client's order is to buy (sell). Second, the winning dealer may fulfill the client's order by trading on the market, where our model permits two periods of on-market trading. Losing dealers are also free to trade on the market.

In equilibrium, a losing dealer trades on the market in roughly two distinct ways. If he is long (short) while the client's order is to buy (sell), then he might provide liquidity to the winning dealer by selling (buying), which reduces the winning dealer's trading costs. On the other hand, he might front-run the winning dealer by buying (selling) initially and subsequently reversing those trades, which could increase the winning dealer's trading costs.[1] Both types of trading are profitable for

[1]This behavior is what we refer to as "front-running," and it comports with its legal definition whereby trading

the losing dealer. Anticipating this trading behavior, dealers' bids account for both (*i*) the trading costs they would incur if they won, and (*ii*) the opportunity cost consisting of the profits they would obtain if they lost.

Our main results pertain to *RFQ policies*, which specify how the number of dealers she contacts, the signal she provides, and the reserve prices she specifies will depend on characteristics of the client's desired trade. One question is what information the client ought to provide to the dealers while issuing the RFQ. The number of dealers contacted might already signal the client's desired trade (for instance, if the client's policy is to contact two dealers more frequently when buying than when selling). But other information might be provided on top of that. One extreme is the case of *no disclosure*. In our model, the prior distribution of desired client trades has two-point support: the client will wish either to buy or to sell a fixed amount. Hence, no disclosure is equivalent to asking the dealers to "make a two-sided market" (and saying nothing else about her desired trade).

Can the client do better by revealing additional information? Such additional information would have an effect because it facilitates a losing dealer's ability to trade on the market, potentially increasing the scope for both harmful front-running and helpful liquidity provision. In general, the client provides the dealers with a signal about her desired trade, as in Bayesian persuasion. For example, one possibility is the opposite extreme: the case of *full disclosure*, in which the client fully reveals her desired trade (in other words, asking for a "one-sided market"). We obtain a strong result: no disclosure is unambiguously optimal among *all* information structures. Notably, this result is in line with common industry practice, where additional information is rarely volunteered at the RFQ stage.

To understand the optimality of no disclosure, note first that information design makes a difference only to a dealer who is contacted but does not win the RFQ, and thus only when the client contacts both dealers. Having observed this, the intuition can be explained by focusing on the case in which both dealers are initially long. In the full-disclosure regime, the losing dealer can condition his first-period trade on the direction of the client's order. He uses this ability to do some amount of front-running in both cases: (*i*) if the client's order is to buy, he buys in the first period then sells back a larger amount in the second period (providing liquidity on net), (*ii*) if the client's order is to sell, he sells in the first period then buys back the same amount in the second period (providing no liquidity on net). We now contrast this with the no-disclosure regime, where the losing dealer's first-period trade *cannot* condition on the direction of the client's order and must reflect a compromise between those two cases. This reduces the amount of front-running. Moreover, it does *not* reduce the amount of liquidity provided: (*i*) neither in the first period, because the losing dealer had not been providing liquidity in the first period under full disclosure, (*ii*) nor in the second period, because the direction of the order is anyway revealed prior to the second period through the winning dealer's equilibrium first-period trades. By reducing front-running without reducing liquidity provision, the elimination of disclosure creates two effects: (*i*) the winning dealer's trading costs decline, and (*ii*) the losing dealer's profits (i.e., the winning dealer's opportunity cost) decline.

on "non-public market information concerning an imminent block transaction" is prohibited (*cf.* FINRA Rule 5270).

Both effects lead dealers to bid more aggressively, reducing the cost of procurement for the client.

Finally, we answer the related question about how many dealers the client ought to contact. It is not always optimal to contact all available dealers. In particular, this feature obtains despite the fact that our model does not feature exogenous search frictions, as assumed by Duffie, Gârleanu and Pedersen (2005) and much of the follow-on literature.

That the client can benefit from restricting participation in her procurement auction contrasts with conventional auction models, where the presence of additional bidders tends to unambiguously benefit the auctioneer. The reason is that these dealers interact not only in the auction but also after the auction, on the market. In particular, dealers who are contacted but do not win will front-run on the market, raising the winning dealer's trading costs; it follows that an additional competitor in the auction might actually induce a dealer to bid *less aggressively*. The client optimally contacts only a single dealer when this risk of front-running is highest: when she needs to buy (or, respectively, sell) and when there is a sufficiently large prior probability that the dealers are initially long (or, respectively, short). This intuition also relates to our result about the optimality of no disclosure: this information policy is optimal precisely because it reduces the scope for front-running by the losing dealer, thereby inducing more aggressive bids.

Applications. Our model is relevant for a variety of settings and asset classes. Indeed, the client's RFQ process could be implemented informally, via traditional "voice" trading—in which case our results rationalize some behaviors observed in practice. Alternatively, it could be implemented via formal RFQ protocols, such as those in use on swap execution facilities (SEFs)—in which case our results have implications for how these protocols should be designed. Furthermore, the on-market trading of our model could refer to a centralized exchange (as exists in, e.g., equities trading).[2] Alternatively, for asset classes that trade over the counter, it could refer to an inter-dealer market (as exists in, e.g., bond trading).

Just as it is central to our analysis, front-running is often cited as an important consideration by market participants in these settings. For example, the CFTC had once proposed a rule that would mandate RFQs to be sent to no fewer than five dealers. (The rule was since adopted with the requirement reduced to three.) Comment letters from Bloomberg, BlackRock, MetLife, Barclays, Morgan Stanley, and others objected, claiming that it is sometimes advantageous to contact fewer dealers—precisely because doing so limits the front-running and information leakage that we model:

> Several commenters specifically noted that the five market participant requirement may result in increased spreads for participants because non-executing market participants in the RFQ could "front run" the transaction in anticipation of the executing market participant's forthcoming and offsetting transactions. (CFTC, 2013)

Our analysis is very much in line with this concern about front-running. Indeed, our results lend support to this argument against such rules mandating a trader to contact a minimum number of

[2] Such a centralized exchange is sometimes referred to as the "downstairs" market. In contrast, block trades (such as that between the client and the winning dealer in our model) are sometimes referred to as occurring "upstairs."

potential counterparties.

Related literature. Our model assembles several moving parts, and each of those parts relates to a separate literature.

In the procurement auction that the client conducts, dealers' bids are affected by subsequent on-market trading that they anticipate. Related is the literature on auctions with externalities, which is motivated by settings with downstream interaction among the bidders (Jehiel, Moldovanu and Stacchetti, 1996; Jehiel and Moldovanu, 1996, 2000). Also related, Dworczak (2020) considers mechanism design in the presence of aftermarkets.

When soliciting bids, the client chooses what information to provide. Milgrom and Weber (1982) analyze auctions with affiliated values in which the auctioneer can publicly disclose information prior to bidding. According to their classic *linkage principle*, the auctioneer optimally discloses everything she knows. In sharp contrast, in our different setting, we find that the client optimally discloses nothing about her intended trade. Bergemann and Pesendorfer (2007) analyze auctions with independent private values in which the auctioneer can privately disclose information to each bidder prior to bidding.[3] Methodologically, our analysis of the client's information design problem uses tools from the Bayesian persuasion literature (Kamenica and Gentzkow, 2011).

There is also the on-market trading that follows the auction. In some cases, losing dealers front-run in a way and for reasons reminiscent of the literature on predatory trading (Brunnermeier and Pedersen, 2005; Carlin, Lobo and Viswanathan, 2007; Sannikov and Skrzypacz, 2016). This behavior raises trading costs for the winning dealer and hence the client's cost of procurement. The client can, however, influence the extent of front-running through the information she provides and the number of dealers she contacts.

Finally, the client's attempt to select a counterparty in our model connects to the literature on counterparty search. In the canonical model of Diamond (1982), and in its adaptations to over-the-counter markets (Duffie, Gârleanu and Pedersen, 2005, 2007), search is modeled as an exogenous and random matching process. The parties meet randomly in pairs and bargain over the terms of trade. We contribute to this literature by micro-founding and endogenizing the client's search for a suitable dealer. In our model, the client conducts a batch search by soliciting RFQs from potentially multiple dealers. What may prevent her from contacting all available dealers in equilibrium is not a physical barrier, but adverse effects due to front-running by a dealer who is contacted but not chosen.

The most closely related papers are those that similarly micro-found search frictions in the context of counterparty search. In Burdett and O'Hara (1987), a trader does not contact all potential counterparties because of reduced-form front-running concerns. In Keim and Madhavan (1996), this is because contacts are costly. In Zhu (2012), this is because search is sequential with expiring offers. In Riggs, Onur, Reiffen and Zhu (2020), this is because contacts are costly and

[3] They assume that the auctioneer has full control over each bidder's information structure. Additionally, others have investigated settings in which bidders are endowed with private information of their own, so that the auctioneer has only limited control over what the bidders know (Ező and Szentes, 2007; Li and Shi, 2017).

due to the presence of a winner's curse.[4] These papers differ from our model in a variety of ways. Perhaps most significantly, none of them allow the trader to vary the information available to potential counterparties at the RFQ stage; hence, they do not address the questions of information design that we study.

2 Model

We begin with a formal description of the model. We next discuss some of the key modeling choices, then introduce the parametric assumptions used in our baseline analysis.

2.1 Setup

Players. There are three players: a client and two dealers. The dealers are labelled A and B, and it is useful to define $\mathcal{I}_0 = \emptyset$, $\mathcal{I}_1 = \{A\}$, and $\mathcal{I}_2 = \{A, B\}$. The client realizes a trading need of size $\bar{s}$ shares. Depending on whether this is to buy or to sell, we say that the client's trading need is $s \in \{-\bar{s}, \bar{s}\}$. The prior probability that the client seeks to buy (i.e., that $s = \bar{s}$) is $\phi_0 \in (0, 1)$.

Timing and information sets. First, the client observably commits to a *request-for-quote (RFQ) policy.* Such a policy consists of a finite realization space Σ together with a profile of distributions $(\pi_{s'})_{s' \in \{-\bar{s}, \bar{s}\}}$ over $\Sigma \times \{0, 1, 2\} \times \mathbb{R}^2$. The interpretation is that, given the realized s, a realization $(\sigma, M, \bar{b}) \sim \pi_s$ will be drawn. At that point, the dealers $i \in \mathcal{I}_M$ will be contacted, informed of σ, and invited to participate in a second-price auction with M bidders and with reserve prices $\bar{b} = (\bar{b}_{-\bar{s}}, \bar{b}_{\bar{s}})$.

Second, Nature draws the client's trading need $s \in \{-\bar{s}, \bar{s}\}$, where again, ϕ_0 is the prior probability of $\bar{s}$. Nature also draws a vector of initial dealer inventories $(e^A, e^B) \in \{-\bar{e}, \bar{e}\} \times \{-\bar{e}, \bar{e}\}$. Both dealers commonly observe the entire vector of realized inventories. We parameterize the joint distribution of (e^A, e^B) so that (i) $\rho \in (-1, 1)$ is the correlation of e^A and e^B, and (ii) for each dealer i, $\psi \in \left(\max\{0, -\frac{\rho}{1-\rho}\}, \min\{1, \frac{1}{1-\rho}\}\right)$ is the marginal probability that $e^i = \bar{e}$. To that end:

$$(e^A, e^B) = \begin{cases} (\bar{e}, \bar{e}) & \text{w.p. } \psi[1 - (1 - \psi)(1 - \rho)] \\ (\bar{e}, -\bar{e}) & \text{w.p. } \psi(1 - \psi)(1 - \rho) \\ (-\bar{e}, \bar{e}) & \text{w.p. } \psi(1 - \psi)(1 - \rho) \\ (-\bar{e}, -\bar{e}) & \text{w.p. } (1 - \psi)[1 - \psi(1 - \rho)] \end{cases}$$

Third, the client follows through on the RFQ policy to which she previously committed. That is, $(\sigma, M, \bar{b}) \sim \pi_s$ is drawn and observed by each dealer $i \in \mathcal{I}_M$. Such a realization is called an *RFQ*.

[4] Related, Bulow and Klemperer (2002) show that an auctioneer might benefit from restricting participation in common-value or almost-common-value auction settings because of the presence of a similar winner's curse. An auctioneer might also benefit from restricting participation if it is costly for bidders to learn their values (Levin and Smith, 1994) or to prepare their bids (Menezes and Monteiro, 2000).

Fourth, each dealer $i \in \mathcal{I}_M$ submits a bid. Such a bid is a vector $b^i = (b^i_{s'})_{s' \in \{-\bar{s}, \bar{s}\}}$, where $b^i_{s'}$ represents the smallest commission that dealer i will accept for facilitating a trade of s' shares.

Fifth, a second-price auction with reserve is held. Let $b_s^{(m)}$ denote the mth order statistic among $(b^i_s)_{i \in \mathcal{I}_M}$. The winning dealer is chosen uniformly at random from $\{i \in \mathcal{I}_M : b^i_s = \min(\bar{b}_s, b_s^{(1)})\}$. If there is a winner, then the auction also determines a procurement cost $c \equiv \min(\bar{b}_s, b_s^{(2)})$. The winning dealer then observes s, while any losing dealers observe only the identity of the winner. If there is no winner, the game ends.[5]

Sixth, two periods of on-market trading occur. For the first such period, each dealer $i \in \{A, B\}$ simultaneously submits a market order to buy $x^i_1 \in \mathbb{R}$ shares. These first-period trades are consummated at the price $p_1 = p_0 + \theta(x^A_1 + x^B_1)$. Having observed p_1, each dealer $i \in \{A, B\}$ then simultaneously submits a market order to buy $x^i_2 \in \mathbb{R}$ shares. These second-period trades are consummated at the price $p_2 = p_1 + \theta(x^A_2 + x^B_2)$. Dealers make these trading decisions subject to the following constraints: (i) if dealer i wins, then $e^i + x^i_1 + x^i_2 - s \in [-\bar{e}, \bar{e}]$; (ii) if dealer i does not win, then $e^i + x^i_1 + x^i_2 \in [-\bar{e}, \bar{e}]$; (iii) if dealer i is not contacted, then $x^i_1 = 0$. To simplify the proofs, we also assume the constraint $x^i_1 \in [-\bar{s}, \bar{s}]$, which never binds and is assumed only to reduce the number of cases that need to be formally checked.[6]

Seventh, the client pays the winning dealer $c + sp_0$ in exchange for s shares. Outstanding positions are then liquidated for a dividend of p_0.

Payoffs. If there is no winner, the client's payoff is $\bar{u}$. If there is a winner, the client's payoff is $-c$. If dealer i wins, his payoff is $c + (e^i + x^i_1 + x^i_2 - s)p_0 - p_1 x^i_1 - p_2 x^i_2$. If dealer i does not win, his payoff is $(e^i + x^i_1 + x^i_2)p_0 - p_1 x^i_1 - p_2 x^i_2$. All players are risk-neutral expected utility maximizers.

2.2 Remarks

Our formulation of client's RFQ policy generalizes how communication is modeled in the Bayesian persuasion literature (Kamenica and Gentzkow, 2011). In that literature, a sender, who will privately observe her type $s \in \mathcal{S}$, commits to a signal realization space Σ together with a profile of distributions $(\pi_{s'})_{s' \in \mathcal{S}}$ over Σ. In our formulation, the client likewise commits to a policy that determines the signal she will send—but also the number of dealers she will contact and the reserve prices she will use. A standard interpretation of this commitment assumption is that commitment power may come informally through reputation and repeated interaction.[7] And indeed, clients do

[5] Requiring the game to end here is equivalent to requiring that all subsequent on-market trades by the dealers be for zero shares (i.e., $x^i_1 = x^i_2 = 0$ for $i \in \{A, B\}$). We could allow for nonzero on-market trades when there is no winner; however, no such trading would take place in the equilibrium of any such subgame. Thus, requiring the game to end is a harmless assumption made only for simplicity.

[6] Here is the intuition for why this constraint on first-period trade sizes does not bind in equilibrium. The most the winning dealer would ever need to trade on the market (to satisfy constraints on his final inventory) is $\bar{s}$. Due to price impact, he would not wish to trade more than that amount in total—and certainly not more than that amount in the first period. Given that, the losing dealer also would not wish to trade more than $\bar{s}$ in the first period.

[7] One of our main results is that it is optimal for the client to communicate no information through the signal that she sends (although other information may be communicated through the number of dealers that she contacts). Section 4 shows that this optimality of no disclosure *does not depend* on the client's ability to commit. Thus, allowing

typically have a limited number of dealers with whom they interact repeatedly.

The type of client-dealer interactions that we model are often of a repeated nature in reality. (Indeed, as just discussed, the standard commitment assumption might be motivated by repeated interaction.) Although dynamics could introduce several important and interesting forces, this paper focuses a one-shot version of this interaction because that seems a natural place to start.

We assume that the client uses a second-price auction with reserve. Identical results would be obtained if the auction mechanism were a first-price auction with reserve. However, as in Bertrand competition, we would then need to either (*i*) specify an appropriate tie-breaking rule, or (*ii*) use a model with discrete bidding increments. Our assumption of a second-price auction circumvents these issues. However, this assumption is not completely without loss. Indeed, given that the dealers' initial inventories may be correlated, more complex auction formats could permit full surplus extraction (e.g., Crémer and McLean, 1988). Our analysis rules out such possibilities because we think it is more realistic to focus on standard auction formats.

We assume that dealers are contacted simultaneously. This assumption reflects settings such as SEFs, where it is commonplace and, in some cases mandatory, that multiple dealers be contacted simultaneously.[8] In contrast, there is a sizable literature on sequential search markets, and indeed, other settings may be more aptly thought of in such terms. However, we suspect that the main forces of our model would be at work also in such settings. A related point concerns the source of dealer uncertainty in the model. In sequential search, a dealer's uncertainty about the order in which he has been contacted plays an important role (Zhu, 2012). In modeling batch search, our analysis abstracts away from contact-order uncertainty. Instead, dealers' uncertainty in our model concerns the direction of the client's desired trade. Though our model contains no asymmetric information about the security's fundamental value, uncertainty about the client's trading direction is relevant because markets are imperfectly competitive (i.e., because there is price impact).

We have assumed that dealers have position limits of $\bar{e}$ shares, in the sense that each dealer's final inventory (after both periods of on-market trading and after any trade with the client) must not exceed $\bar{e}$ shares, either long or short. This position limit could be interpreted as either a risk limit or a capital constraint.[9] It is also not important that these position limits are binding at the initial inventories. Indeed, some cases in which there is a given amount of slack at the initial inventories are equivalent within the model to what obtains under a suitable adjustment to $\bar{s}$.

The primary interpretation of a dealer's inventory in our model is the dealer's proprietary position. However, a potential secondary interpretation is the order flow of the dealer's other (unmodeled) clients. Indeed, if these other clients seek to buy (sell), then that would, all else equal,

the client to commit to a policy for determining signals *only strengthens* our result about the optimality of no disclosure.

[8] We also assume that the contacted dealers know how many other dealers have been contacted. This assumption also reflects settings such as RFQs made on SEFs. For descriptions of the market structure and regulation pertaining to swaps trading, see Collin-Dufresne, Junge and Trolle (2020); Riggs, Onur, Reiffen and Zhu (2020).

[9] We additionally assume that dealers have no holding costs for inventories within the position limits. If such holding costs were present, they would create a separate trading motive for the dealers. Hence, this assumption allows us to isolate the trading dynamics that stem from the client's presence.

induce the dealer to bid more aggressively for a selling (buying) order from the focal client—just as a long (short) propriety position would.[10]

What we aim to capture with the assumption that an un-contacted dealer i must set $x_1^i = 0$ is that in practice, a dealer who was not contacted would not be able to predict (and trade in anticipation of) the precise moment of the client's arrival. In contrast, an artifact of the structure of the model is that the timing of the client's arrival is common knowledge among the players. Hence, we impose this constraint to prevent behavior in the model from being driven by this artifact and to instead force it to align with what we would expect to see in practice.

We have assumed that each dealer knows the other's realized initial inventory. Although this assumption of perfect knowledge is extreme (albeit useful for tractability), dealers might be quite familiar with their competitors in practice. Additionally, dealers might obtain signals about competitors' positions by, for example, inspecting published indications of interest, by inspecting trade data, or via communication (e.g., informal backroom chats or so-called "talking your book"). We have also assumed that a dealer's information about its competitor is strictly better than that of the client (who knows only the prior distribution of dealer inventories). This also seems plausible, especially for clients who interact with dealers at a frequency lower than that at which dealers interact with each other.

A natural case of interest is that in which the client has symmetric buying and selling needs (i.e., $\phi_0 = \frac{1}{2}$) and in which the dealers have equal probabilities of being long and short (i.e., $\psi = \frac{1}{2}$). However, there is value in allowing for asymmetric parametrizations. For example, settings where dealers are more likely to be long than short (perhaps because there are additional frictions associated with short positions) are captured by $\psi > \frac{1}{2}$. Likewise, settings where the client is more likely to be buying than selling (e.g., because the client is a pension fund with young members) are captured by $\phi_0 > \frac{1}{2}$.

We assume an exogenous price process, with θ as the coefficient of permanent price impact. This price process resembles, for example, the baseline model of Bertsimas and Lo (1998). Moreover, it could be micro-founded by adding a competitive fringe of long-term investors to the model. Specifically, assume that at each trading period, the aggregate demand of these investors is $Y(p) = \frac{1}{\theta}(p_0 - p)$. Such a downward-sloping demand could stem from several sources, including risk aversion, institutional frictions, or concerns about adverse selection (although, to be clear, our model does not feature asymmetric information about fundamentals). Note also that this aggregate demand depends only on the current price p, so that these traders do not attempt to profit from short-term price swings. Brunnermeier and Pedersen (2005) make precisely the same set of assumptions, which they argue could be appropriate if these long-term traders lacked the information, skills, or time necessary for predicting price changes.

Furthermore, the assumed price process is a deterministic function of the previous period's price and the current period's trades. This is mainly for tractability: adding price shocks would complicate the belief updating of losing dealers.

[10] A *riskless principal trade* is said to occur when a dealer conducts opposing trades with two separate clients.

We have assumed that the client can obtain execution only by trading with a dealer (and cannot trade on the market directly).[11] This assumption is realistic for many institutional investors who commonly lack either the infrastructure, the expertise, or the regulatory clearance to access markets directly. Alternatively, the client's outside option $\bar{u}$ might be interpreted as her expected utility derived from other modes of trading. (We do, however, subsequently specialize the model to the case of $\bar{u} = -\infty$ so as to ensure that the client uses a dealer on path.)

We restrict attention to fixed-price contracts (as opposed to, for example, contracts that condition on the market prices p_1 and p_2). This restriction is not unrealistic, as such arrangements are common in many settings. In addition, fixed-price contracts greatly limit the dimensionality of bids, thereby making the analysis more tractable. Finally, it is natural to focus on fixed-price contracts in settings where all parties are risk neutral. Indeed, if dealers were risk averse, then a motive for conditioning on p_1 and/or p_2 would be to insure the dealer against price risk (which would exist in our model if there were price shocks).[12]

We also restrict attention to situations in which the client awards her entire order to a single dealer (as opposed to splitting the order across dealers). Partly, this is for simplicity: it limits the potential outcomes of the auction. Partly, this is to reflect reality: our discussions with industry participants reveal that the same order is rarely split among competing dealers (although a subsequent order might well go to a different dealer). Moreover, awarding the entire order to a single dealer may in fact be optimal for a variety of reasons: (*i*) share auctions often produce worse outcomes for the auctioneer than corresponding unit auctions (Wilson, 1979), and (*ii*) a dealer who receives part of the order would not internalize the externalities that its trading creates for dealers who receive the balance of the order, increasing procurement costs for reasons similar to the double-marginalization problem in a vertical supply chain (as we show formally in Appendix B).

2.3 Parametric assumptions

Henceforth, we set—without loss of generality—the initial price level to $p_0 = 0$, the coefficient of permanent price impact to $\theta = 1$, and dealers' position limits to $\bar{e} = 1$.

We also assume the client's trade size is $\bar{s} \leq 2$. This is not without loss of generality, but yields conclusions qualitatively similar to what obtains with larger $\bar{s}$.[13]

We also assume that the client experiences infinite disutility from not trading (i.e., $\bar{u} = -\infty$). Although this assumption might appear extreme, what it really means is that our analysis will characterize the cost-minimizing RFQ policy among those ensuring execution with probability one.

[11] For models of endogenous choice between trading on the market and trading with a dealer, see, e.g., Seppi (1990); Lee and Wang (2021).

[12] For an investigation of optimal contracts for principal trading with a risk averse dealer (in a model with price shocks), see Baldauf, Frei and Mollner (2021a,b).

[13] The assumption $\bar{s} \leq 2$ is made for tractability. Larger values of $\bar{s}$ would need to be handled as mathematically separate cases for the reason that additional constraints come into play. For example, with $\bar{s} > 2$, the winning dealer can never fully internalize the client's order, and so must trade at least some amount on the market regardless of his initial position.

3 Equilibrium

We begin by describing our solution concept, a refinement of weak perfect Bayesian equilibrium (WPBE). With that in hand, we solve backward, first deriving outcomes following a given RFQ policy, then deriving an optimal RFQ policy.

3.1 Solution concept

The solution concept is a refinement of WPBE. To explain it, first observe that a WPBE consists of the following elements: (*i*) an RFQ policy for the client, (*ii*) actions for the dealers (i.e., bids and trades), and (*iii*) beliefs for the dealers.

Bidding behavior. The first three aspects of the refinement concern bidding behavior. In a classic second-price auction, there are two payoff-relevant outcomes for each bidder (i.e., winning and losing), and each bidder has a unique dominant strategy—bidding the difference in its values across those outcomes. However, a bidder might not have a dominant strategy if the game were to include not only the auction but also a subsequent strategic interaction that could affect the relative values of winning and losing. But there is an analogue. In particular, if the equilibrium of the subsequent interaction were computed and used to fix the bidders' values for the auction (as by backward induction), then the auction could be analyzed in isolation. Conceiving of the auction in isolation in this way, each bidder has a unique dominant bid, as before. Thus, a natural refinement for the entire game would require that these bids are played in the bidding stage.

As mentioned, in a classic second-price auction, there are two payoff-relevant outcomes for each bidder: winning and losing. But in auctions with externalities, more outcomes are in play. For example, a two-bidder auction with externalities has *three* payoff-relevant outcomes: either bidder A wins, bidder B wins, or neither wins. Thus, there is again no dominant strategy. However, if such an auction's reserve price were non-binding (so that the outcome in which neither wins does not occur in equilibrium), then there would again be an analogue: bidder A should bid the difference in its values across the outcomes in which it wins and in which bidder B wins. Thus, a natural refinement would require that these bids are played under non-binding reserves.

We also require bidders to use symmetric bidding strategies.

Beliefs. The final aspects of the refinement concern dealer beliefs. Although the winning dealer observes whether the client seeks to buy or sell, the other dealer does not observe this directly and must instead infer from what he can observe. Our refinement applies to how this other dealer infers from the winning dealer's first-period trades.

We focus on equilibria in which these beliefs have a step-function structure. To state this formally, suppose (without loss of generality) that dealer A is the winning dealer, and let $\mu_2^B(x_1^A)$ denote the probability that $s = \bar{s}$ under B's beliefs as a function of A's first-period trade. We require this to be a function that jumps from zero to one: (*i*) for all $x \in [-\bar{s}, \bar{s}]$, $\mu_2^B(x) \in \{0, 1\}$, and (*ii*) for all $x' < x''$, $\mu_2^B(x') = 1 \implies \mu_2^B(x'') = 1$ and $\mu_2^B(x'') = 0 \implies \mu_2^B(x') = 0$. One implication of this requirement is that we focus on separating equilibria in which the winning dealer's first-period trade fully reveals the realized s to the other dealer.

We also require these beliefs to satisfy an analogue of the intuitive criterion (Cho and Kreps, 1987). To explain, again suppose (without loss of generality) that dealer A is the winning dealer. What can dealer B believe about s following an out-of-equilibrium choice for x_1^A? Fix an equilibrium. For $s' \in \{-\bar{s}, \bar{s}\}$, let $C_*^A(s')$ represent A's equilibrium trading costs given s', and let $C^A(s', x_1^A, \mu_2^B)$ represent A's trading costs from x_1^A and the equilibrium x_1^B, together with the second-period trades in the equilibrium of the game continuing from (x_1^A, x_1^B) given s' and given that B believes $s = \bar{s}$ with probability μ_2^B.[14] We require that for all out-of-equilibrium x_1^A, neither of the following pairs of conditions holds:

$$C_*^A(\bar{s}) < \min_{\mu_2^B \in [0,1]} C^A(\bar{s}, x_1^A, \mu_2^B) \quad \text{and} \quad C_*^A(-\bar{s}) > C^A(-\bar{s}, x_1^A, 0)$$

$$C_*^A(-\bar{s}) < \min_{\mu_2^B \in [0,1]} C^A(-\bar{s}, x_1^A, \mu_2^B) \quad \text{and} \quad C_*^A(\bar{s}) > C^A(\bar{s}, x_1^A, 1)$$

After stating Lemma 1, we provide an example to illustrate this criterion and to review the motivation that Cho and Kreps (1987) provide for it.

Henceforth, we use *equilibrium* to refer to this refinement of WPBE.

3.2 Contacting one dealer

We begin by analyzing continuation equilibrium in subgames following RFQs that contact one dealer. Recall that an RFQ policy is defined by a signal realization space Σ and distributions $(\pi_{s'})_{s' \in \{-\bar{s}, \bar{s}\}}$ over $\Sigma \times \{0, 1, 2\} \times \mathbb{R}^2$. Fix such an RFQ policy. Fix, moreover, some RFQ $(\sigma, M, \bar{b})$ with $M = 1$. It suffices to focus on RFQs that occur on path, and so we assume $\pi_{-\bar{s}}(\sigma, M, \bar{b}) + \pi_{\bar{s}}(\sigma, M, \bar{b}) > 0$. It is also useful to define

$$\phi = \frac{\phi_0 \pi_{\bar{s}}(\sigma, M, \bar{b})}{(1 - \phi_0)\pi_{-\bar{s}}(\sigma, M, \bar{b}) + \phi_0 \pi_{\bar{s}}(\sigma, M, \bar{b})}$$

as the posterior probability of $s = \bar{s}$ induced by this RFQ. Note that $\phi \in [0, 1]$ and that it need not coincide with the prior ϕ_0.

Lemma 1. *In the class of RFQs that contact $M = 1$ dealer and guarantee execution with probability one, the minimum expected procurement cost is $\hat{c}_1 = \frac{3\bar{s}^2}{4}$. It is achieved by RFQs featuring reserve prices $\bar{b} = (\frac{3\bar{s}^2}{4}, \frac{3\bar{s}^2}{4})$.*

To prove Lemma 1, we begin by deriving the unique equilibrium actions in the subgame following any RFQ that contacts one dealer. We sketch that derivation below. Then at the end of this section,

[14] The main subtlety is in defining $C^A(s, x_1^A, \mu_2^B)$. Technically, the equilibrium that we have fixed does not specify what second-period trades would be under out-of-equilibrium beliefs μ_2^B. Nevertheless, the structure of the model allows these to be uniquely computed in a straightforward manner.

we explain how the lemma's claim follows.

Continuation equilibrium. Because both dealers observe the entire vector (e^A, e^B), the four possible realizations of that vector can be analyzed separately. Let us focus on the case of $(e^A, e^B) = (1, -1)$. Suppose that dealer B's beliefs in this case are that $s = \bar{s}$ with probability

$$\mu_2^B = \begin{cases} 1 & \text{if } -\frac{\bar{s}}{6} \leq x_1^A \leq \bar{s} \\ 0 & \text{if } -\bar{s} \leq x_1^A < -\frac{\bar{s}}{6} \end{cases} \tag{1}$$

In words, if dealer A's first-period trade is above (below) the cutoff $-\frac{\bar{s}}{6}$, then dealer B believes with certainty that the client is a buyer (seller). For an informal derivation of the unique on-path actions, consider two subcases:

- First, suppose $s = \bar{s}$. Ignoring the constraints on final inventory (which will not bind in the equilibrium), dealers A and B respectively minimize

  $$x_1^A x_1^A + (x_1^A + x_2^A + x_2^B)x_2^A$$
  $$(x_1^A + x_2^A + x_2^B)x_2^B,$$

  leading to $x_2^A = x_2^B = -\frac{x_1^A}{3}$.[15] Inducting backward, we obtain $x_1^A = 0$, so that $x_2^A = x_2^B = 0$ on path.

  Plugging in these trades, dealer A incurs no trading costs if he wins, so that it is optimal—and in fact required by the refinement described in Section 3.1—that he bid $b_{\bar{s}}^A = 0$.

- Second, suppose $s = -\bar{s}$. Assuming that $x_2^A = -\bar{s} - x_1^A$ (which ensures that dealer A's final inventory just meets the constraint $e^A + x_1^A + x_2^A - s \leq 1$) and ignoring all other constraints on final inventory, dealers A and B respectively minimize

  $$x_1^A x_1^A + (-\bar{s} + x_2^B)(-\bar{s} - x_1^A)$$
  $$(-\bar{s} + x_2^B)x_2^B,$$

  leading to $x_2^B = \frac{\bar{s}}{2}$. Inducting backward, we obtain $x_1^A = -\frac{\bar{s}}{4}$.

  Plugging in these trades, dealer A incurs trading costs of $\frac{7\bar{s}^2}{16}$ if he wins, so that it is optimal—and in fact required by the refinement described in Section 3.1—that he bid $b_{-\bar{s}}^A = \frac{7\bar{s}^2}{16}$.

The client's cost of procurement. Next, we explain how the lemma's claim follows from what we have derived about equilibrium bids. The client's procurement cost is determined by a second-price procurement auction. In this case where only one dealer is contacted, the reserve is what sets the price. Of course, a constraint is that execution does not occur when the dealer's bid exceeds the

[15] Note that, although dealer B cannot directly condition x_2^B on the realized s, he can do so *indirectly* because this is a separating equilibrium in the sense that x_1^A reveals s.

reserve. To ensure execution with probability one, the client's reserve must be at least the dealer's bid in the worst case. If the client wishes to sell, the worst case is $(e^A, e^B) = (1, 1)$. As we specify in the proof of Lemma 1, $b^A_{-\bar{s}} = \frac{3\bar{s}^2}{4}$ in this case. (By symmetry, this is also the worst-case value for $b^A_{\bar{s}}$.) Therefore, the expected procurement cost resulting from an RFQ that contacts one dealer, induces a belief that ϕ is the probability of $s = \bar{s}$, and guarantees execution with probability one results is at least

$$\hat{c}_1 = \phi\frac{3\bar{s}^2}{4} + (1-\phi)\frac{3\bar{s}^2}{4} = \frac{3\bar{s}^2}{4},$$

as claimed by Lemma 1. Moreover, the RFQ that uses reserve prices $\bar{b} = (\frac{3\bar{s}^2}{4}, \frac{3\bar{s}^2}{4})$ both ensures execution with probability one and achieves the cost $\hat{c}_1$.

Note that $\hat{c}_1$ does not depend on the probability of $s = \bar{s}$ that the RFQ induces (which we have labelled ϕ). This is for two reasons. First, the reserve prices specified by Lemma 1 are constant in ϕ. Intuitively, this is because there is no role for information design when $M = 1$: the contacted dealer submits a separate quote for each state, and he will moreover learn the state before he has to trade. Second, these reserve prices are both $\frac{3\bar{s}^2}{4}$. Intuitively, this is due to the model's symmetric structure. Thus, although $\hat{c}_1$ can be thought of as a weighted average of the two (with weight ϕ on $\bar{b}_{\bar{s}}$), the weight does not matter. As we will see, this changes when two dealers are contacted: the analogous quantity $\hat{c}_2$ will be a non-constant (and in fact non-linear) function of ϕ, which creates a role for information design.

Likewise, $\hat{c}_1$ also does not depend on ρ and ψ, which govern the distribution of (e^A, e^B). This is because the client's cost is set by her reserve prices, and because the reserve prices specified by Lemma 1 are driven by the worst case. Although (ρ, ψ) affect the distribution over the various cases for (e^A, e^B), they do not affect equilibrium outcomes in those cases—and in particular do not affect the worst case. As we will see, this also changes when two dealers are contacted: the client's cost may then be set by the losing dealer's bid, and thus no longer driven only by the worst case.

The intuitive criterion. Finally, let us provide an example to illustrate what role the intuitive criterion (Cho and Kreps, 1987) plays in our equilibrium selection. As above, focus on the case of $(e^A, e^B) = (1, -1)$. There is another WPBE in which dealer B's beliefs entail a lower cutoff than that in (1)

$$\mu^B_2 = \begin{cases} 1 & \text{if } -\frac{\bar{s}}{2} < x^A_1 \leq \bar{s} \\ 0 & \text{if } -\bar{s} \leq x^A_1 \leq -\frac{\bar{s}}{2} \end{cases} \tag{2}$$

and in which dealer A sells more in the first period when $s = -\bar{s}$—in particular, selling just enough to meet the cutoff: $x^A_1 = -\frac{\bar{s}}{2}$. The intuition is that when $s = -\bar{s}$, A's inventory constraint binds, and he must sell on the market. If B believes that $s = -\bar{s}$, then in the second trading period, he provides liquidity to A by buying on the market. And because A would like B to provide this liquidity to him, $x^A_1 = -\frac{\bar{s}}{2}$ is set at the cutoff required for B to believe $s = -\bar{s}$. If a higher choice for x^A_1 (e.g., $-\frac{\bar{s}}{4}$, as in Lemma 1) would induce B to provide liquidity in the second period, then A would prefer that. Intuitively, that outcome would allow A to sell only a minority of the necessary amount in the first period when B is not providing liquidity, while waiting to sell the majority until the second period when B is providing liquidity. However, this outcome is incompatible with the beliefs (2). Under (2), A must instead settle for selling the smallest amount in the first period that does induce B to provide liquidity (i.e., $x_1^A = -\frac{\bar{s}}{2}$).

However, this alternative WPBE fails the version of the intuitive criterion described in Section 3.1. Indeed, in terms of the notation introduced there, A's equilibrium trading costs are $C_*^A(\bar{s}) = 0$ and $C_*^A(-\bar{s}) = \frac{\bar{s}^2}{2}$. Then consider the out-of-equilibrium choice $x_1^A = -\frac{\bar{s}}{4}$. We can compute $\min_{\tilde{\mu}_2^B \in [0,1]} C^A(\bar{s}, -\frac{\bar{s}}{4}, \tilde{\mu}_2^B) = C^A(\bar{s}, -\frac{\bar{s}}{4}, 0) = \frac{3\bar{s}^2}{64} > C_*^A(\bar{s}) = 0$ and $C^A(-\bar{s}, -\frac{\bar{s}}{4}, 0) = \frac{7\bar{s}^2}{16} < \frac{\bar{s}^2}{2}$. Intuitively, if $s = \bar{s}$, then the best that A could expect to achieve from a choice of $x_1^A = -\frac{\bar{s}}{4}$ is a trading cost of $\frac{3\bar{s}^2}{64}$ (achieved if B believes $s = \bar{s}$ with probability zero), which is strictly greater than his equilibrium trading cost of zero.[16] Thus, it would be difficult to make sense of this deviation if $s = \bar{s}$. Could the deviation make sense if $s = -\bar{s}$? Suppose $s = -\bar{s}$ and A deviates to $x_1^A = -\frac{\bar{s}}{4}$, hoping that—since this deviation would not make sense if $s = \bar{s}$—the deviation will induce B to believe $s = -\bar{s}$. If the deviation does indeed induce B to believe this, then the deviation would make sense: A's trading cost would be $\frac{7\bar{s}^2}{16}$ (as in Lemma 1's equilibrium), which is strictly less than his trading cost in this equilibrium of $\frac{\bar{s}^2}{2}$. Thus, we have an argument that—contrary to the beliefs (2)—dealer B should infer from a deviation to $x_1^A = -\frac{\bar{s}}{4}$ that $s = -\bar{s}$.

3.3 Contacting two dealers

We now proceed analogously to analyze continuation equilibrium in subgames following RFQs that contact two dealers. To do so, fix an RFQ policy. Fix also some RFQ $(\sigma, M, \bar{b})$ with $M = 2$. As before, assume without loss of generality that $\pi_{-\bar{s}}(\sigma, M, \bar{b}) + \pi_{\bar{s}}(\sigma, M, \bar{b}) > 0$, and define

$$\phi = \frac{\phi_0 \pi_{\bar{s}}(\sigma, M, \bar{b})}{(1-\phi_0)\pi_{-\bar{s}}(\sigma, M, \bar{b}) + \phi_0 \pi_{\bar{s}}(\sigma, M, \bar{b})}$$

as the posterior probability of $s = \bar{s}$ induced by this RFQ.

Lemma 2. *In the class of RFQs that contact $M = 2$ dealers, induce beliefs ϕ, and guarantee execution with probability one, the minimum expected procurement cost is a convex and differentiable function $\hat{c}_2(\phi)$ that satisfies $\hat{c}_2(\frac{1}{2}) < \hat{c}_1$. It is achieved by RFQs featuring reserve prices $\bar{b} = (\bar{s}^2, \bar{s}^2)$.*

The client's procurement cost is determined by a second-price procurement auction. In this case where two dealers are contacted, it could in principle be set by either the reserve price or the losing dealer's bid. What would transpire if the reserves were low enough to occasionally set the price? Given our restriction to symmetric bidding strategies, it can be shown that the continuation equilibrium would entail mixed bidding strategies, in which with positive probability, neither dealer would meet the reserve. Thus, to ensure execution with probability one, the reserves must be high

[16] When B believes that $s = \bar{s}$ with probability $\tilde{\mu}_2^B$, he chooses $x_2^B = \frac{(8-7\tilde{\mu}_2^B)\bar{s}}{4(4-\tilde{\mu}_2^B)}$. If we actually have $s = \bar{s}$ and $x_1^A = -\frac{\bar{s}}{4}$, then A best responds with $x_2^A = \frac{(3\tilde{\mu}_2^B-2)\bar{s}}{4(4-\tilde{\mu}_2^B)}$, leading to a total trading cost of $\frac{(3-2\tilde{\mu}_2^B)(1+\tilde{\mu}_2^B)\bar{s}^2}{4(4-\tilde{\mu}_2^B)}$. This objective is minimized at $\tilde{\mu}_2^B = 0$, where it evaluates to $\frac{3\bar{s}^2}{64}$.

enough to never set the price. To prove Lemma 2, we therefore begin by analyzing equilibria of subgames following RFQs that contact two dealers and in which the reserves are sufficiently high in this sense. We discuss that derivation, then explain how the lemma's claim follows.

Continuation equilibrium. In the previous section, only one dealer was contacted, and the other dealer was unable to trade in the first period. Here, both dealers are contacted, and not only the winner but also the loser may trade in the first period. In some cases, the losing dealer uses this ability to front-run the winner's trades. To see this front-running, focus on the case of $(e^A, e^B) = (1, 1)$, where both dealers begin long. As we show in the proof, the losing dealer sells in the first period; this pays off for him if $s = -\bar{s}$, in which case he buys back in the second period and nets a profit. Hence, the amount that he sells in the first period depends on his beliefs. If $\phi = 1$, so that he is sure of $s = \bar{s}$, then there is no scope to front-run, and he in fact does not trade. But as ϕ decreases, he gradually sells more in the first period (ultimately selling $\frac{\bar{s}}{3}$ if $\phi = 0$).

To explain how bidding behavior is pinned down, let us continue to focus on the case of $(e^A, e^B) = (1, 1)$ and $s = -\bar{s}$. Once trading behavior has been characterized, we can compute dealer A's trading costs in each of the three potential auction outcomes: (*i*) if he wins, (*ii*) if B wins, and (*iii*) if neither wins. However, if the reserve price $\bar{b}_{-\bar{s}}$ is sufficiently high, then outcome (*iii*) becomes irrelevant. Given that, the refinement described in Section 3.1 requires A's bid to be determined by the difference in his value across outcomes (*i*) and (*ii*). Having derived this bid, we can then fill in what it means for the reserve price to be sufficiently high in this sense: $\bar{b}_{-\bar{s}}$ must be at least this difference. Choosing $\bar{b} = (\bar{s}^2, \bar{s}^2)$, as in Lemma 2, satisfies not only this constraint but also analogous constraints for other realizations of s and (e^A, e^B).

The client's cost of procurement. To compute $\hat{c}_2$, the expected procurement cost under $\bar{b} = (\bar{s}^2, \bar{s}^2)$—or any other reserves high enough that they never set the price—we simply calculate a weighted average of the losing bids. The weights are dictated by the parameters ψ and ρ, which govern the distribution of (e^A, e^B), as well as ϕ, which captures the distribution of s conditional on the RFQ.

In addition to altering these weights, ϕ also affects the bids themselves. The reason is that the losing dealer, not having observed the realized s, relies on these beliefs to select his first-period trade.

To build intuition for how $\hat{c}_2(\phi)$ and $\hat{c}_1$ compare, consider first the case of $(e^A, e^B) = (1, 1)$, in which both dealers begin long. We show in the proof of Lemma 2 that if $\phi = 0$, so that the dealers are certain the client is a seller, then each dealer i bids $b^i_{-\bar{s}} = \bar{s}^2$—in contrast, as previously mentioned, $\frac{3\bar{s}^2}{4}$ is the analogous bid when only one dealer is contacted. The bid is larger here for two reasons. First, the losing dealer's front-running raises the winning dealer's trading costs, so dealers demand more compensation. Second, there is now an additional opportunity cost of winning—namely, the potential for profitable front-running that would exist if the other dealer were to win. It follows from this that if the dealers are likely to begin long (i.e., $\psi \approx 1$), then $\hat{c}_2(0) > \hat{c}_1$. What this means is that for RFQs that reveal the client to be a seller, she is better off contacting only a single dealer. Intuitively, in that case little is gained from having contacted a second dealer (because both are likely long); yet much is lost (because the loser's front-running leads to less aggressive bidding). On the other hand, Lemma 2 says that $\hat{c}_2(\frac{1}{2}) < \hat{c}_1$ (regardless of ψ and ρ). What this means is that for RFQs that imply the client is equally likely a seller or a buyer, she gains on net from inducing competition among the dealers.

3.4 Optimal RFQ policies

The previous sections derived continuation outcomes following any RFQ. We now seek to induct backward so as to obtain the optimal RFQ policy. This problem is potentially complex because there is a large and rich set of such policies to optimize over. How many dealers should the client contact—always one, always two, or should she randomize in some way? What signals should she provide about her desired trade—full disclosure, no disclosure, or something intermediate? How should she treat buying and selling—symmetrically or asymmetrically? Perhaps surprisingly, the model yields sharp answers to these questions.

3.4.1 Optimality of no disclosure

Depending on the RFQ policy, the number of dealers contacted may be an informative signal about the client's desired trade. However, additional information can be provided beyond that. One extreme is the case of *full disclosure*, in which the RFQ fully reveals s. This can be operationalized by defining $\Sigma = \{\texttt{Sell}, \texttt{Buy}\}$, defining $\pi_{-\bar{s}}$ to attach probability one to RFQs of the form $(\texttt{Sell}, M, \bar{b})$, and defining $\pi_{\bar{s}}$ to attach probability one to RFQs of the form $(\texttt{Buy}, M, \bar{b})$. The opposite extreme—the case of *no disclosure*—would be an RFQ policy that reveals nothing beyond what is already implied by the number of contacted dealers—neither via the arbitrary signal σ nor via the reserve prices $\bar{b}$. One implication of the following result is that no disclosure is in fact optimal.

Proposition 3. *There exists an optimal RFQ policy with the following properties: (i) Σ is a singleton $\{\sigma_0\}$; and (ii) the distributions $(\pi_{s'})_{s' \in \{-\bar{s}, \bar{s}\}}$ put positive weight on at most the two RFQs $\big(\sigma_0, 1, (\frac{3\bar{s}^2}{4}, \frac{3\bar{s}^2}{4})\big)$ and $\big(\sigma_0, 2, (\bar{s}^2, \bar{s}^2)\big)$.*

RFQ policies of the form described in the proposition use reserve prices $\bar{b} = (\frac{3\bar{s}^2}{4}, \frac{3\bar{s}^2}{4})$ when $M = 1$ dealers are contacted and $\bar{b} = (\bar{s}^2, \bar{s}^2)$ when $M = 2$. That this is consistent with optimality follows from Lemmas 1 and 2. And given that the client seeks execution with probability one, it clearly suffices to focus on RFQ policies that always contact either one or two dealers (as opposed to zero). So to establish the result, it suffices to show that beginning from an RFQ policy of the form described in the proposition, the client cannot benefit by deviating to a more informative signal structure. Given that $\hat{c}_1$ is a constant, a more informative signal has no effect when only one dealer is contacted. Furthermore, $\hat{c}_2(\phi)$ is convex, so that by standard arguments from the Bayesian persuasion literature (Kamenica and Gentzkow, 2011), the client would actually be worse off under a more informative signal when two dealers are contacted.

To build some intuition for this result, note that the disclosure policy is relevant only through its effect on the information available to the losing dealer in trading period 1. (The winning dealer always learns s, and the loser always infers it after period 1.) Now the question is how the loser's period-1 trade depends on his information about s. With full knowledge of s, the loser would front-run the winner by trading with him in period 1 (while planning to trade against him in period 2). But with imperfect information about s (and hence the direction of the winner's trading), the loser trades a smaller amount in period 1. This reduces the winner's trading costs and leads to more aggressive bidding in the auction. Therefore, the client is better off not revealing anything about s (beyond what is revealed through the number of dealers she contacts).

Notably, this result is in line with common industry practice, where additional information is rarely volunteered at the RFQ stage. For example, clients typically attempt to disguise the direction of their desired trades by asking for two-sided quotes instead of one-sided quotes. Our model rationalizes that behavior as optimal.

3.4.2 Optimal policy for determining the number of dealers to contact

We can therefore restrict attention to RFQ policies of the form described by Proposition 3. For such RFQ policies, and for all $s' \in \{-\bar{s}, \bar{s}\}$, let $q_{s'}$ denote the probability with which two dealers are contacted (so that with complementary probability $1 - q_{s'}$ only one dealer is contacted). The problem therefore reduces to optimizing over $(q_{-\bar{s}}, q_{\bar{s}}) \in [0,1]^2$. The problem therefore reduces to choosing a policy for determining the number of dealers to contact by optimizing over $(q_{-\bar{s}}, q_{\bar{s}}) \in [0,1]^2$. Such a policy has an effect in two ways: (i) given a fixed belief ϕ, it can affect the client's cost, doing so if $\hat{c}_1 \neq \hat{c}_2(\phi)$, and (ii) it can manipulate the beliefs themselves.

To characterize the client's optimal policy, we define $C(\phi)$ as the convex closure of $\min\{\hat{c}_1, \hat{c}_2(\phi)\}$:

$$C(\phi) = \inf\Big\{z \mid (\phi, z) \in co\big(\min\{\hat{c}_1, \hat{c}_2\}\big)\Big\},$$

where $co\big(\min\{\hat{c}_1, \hat{c}_2\}\big)$ denotes the convex hull of the graph of $\min\{\hat{c}_1, \hat{c}_2\}$. By construction, C is the largest convex function that is everywhere weakly less than both $\hat{c}_1$ and $\hat{c}_2(\phi)$. The next result states that $C(\phi_0)$ is a lower bound on the client's procurement cost. What is more, this lower bound is achievable. To describe an RFQ policy that achieves this bound, it is useful to define two cutoffs: $\underline{\phi}$ and $\overline{\phi}$, which are defined precisely to ensure that $C(\phi) = \hat{c}_2(\phi)$ if and only if $\phi \in [\underline{\phi}, \overline{\phi}]$.

Definition 1. Define $\underline{\phi}, \overline{\phi} \in [0,1]$ as follows. If $\hat{c}_2(0) \le \hat{c}_1$, define $\underline{\phi} = 0$; if $\hat{c}_2(1) - \hat{c}_2'(1) \ge \hat{c}_1$, define $\underline{\phi} = 1$; otherwise, define it implicitly as the unique $\underline{\phi} \in (0,1)$ that solves $\hat{c}_2(\phi) - \phi\hat{c}_2'(\phi) = \hat{c}_1$. If $\hat{c}_2(1) \le \hat{c}_1$, define $\overline{\phi} = 1$; if $\hat{c}_2'(0) + \hat{c}_2(0) \ge \hat{c}_1$, define $\overline{\phi} = 0$; otherwise, define it implicitly as the unique $\overline{\phi} \in (0,1)$ that solves $(1-\phi)\hat{c}_2'(\phi) + \hat{c}_2(\phi) = \hat{c}_1$.[17]

Proposition 4. *The following hold:*

[17] To see that this provides a unique definition for $\underline{\phi}$, it suffices to show that $\hat{c}_2(\phi) - \phi\hat{c}_2'(\phi)$ is strictly decreasing on the unit interval. Because its derivative is $-\phi\hat{c}_2''(\phi)$, the conclusion follows from the convexity of $\hat{c}_2(\cdot)$. Analogously, to see that this provides a unique definition for $\overline{\phi}$, it suffices to show that $\hat{c}_2(\phi) + (1-\phi)\hat{c}_2'(\phi)$ is strictly increasing on the unit interval. Because its derivative is $(1-\phi)\hat{c}_2''(\phi)$, the conclusion also follows from the convexity of $\hat{c}_2(\cdot)$.

(i) $C(\phi_0)$ *is a lower bound on the client's cost of procurement.*

(ii) $\underline{\phi} \leq \overline{\phi}$.

(iii) If $\phi_0 \in [\underline{\phi}, \overline{\phi}]$, *then this lower bound is achieved by the RFQ policy defined by* $q_{-\bar{s}} = q_{\bar{s}} = 1$.

(iv) If $\phi_0 \in (0, \underline{\phi})$, *then this lower bound is achieved by the RFQ policy defined by* $q_{-\bar{s}} = \frac{\phi_0(1-\underline{\phi})}{\underline{\phi}(1-\phi_0)}$ *and* $q_{\bar{s}} = 1$.

(v) If $\phi_0 \in (\overline{\phi}, 1)$, *then this lower bound is achieved by the RFQ policy defined by* $q_{-\bar{s}} = 1$ *and* $q_{\bar{s}} = \frac{\overline{\phi}(1-\phi_0)}{\phi_0(1-\overline{\phi})}$.

The intuition for this result can be understood through a tradeoff that contacting an additional dealer entails. On the one hand, the additional dealer may intensify competition among the dealers for the client's business (the *competition effect*). The additional dealer might also be able to provide fulfillment more efficiently (the *sampling effect*). On the other hand, there is a risk that dealers who are contacted but not selected could front-run on the market (the *front-running effect*).

In one set of cases, the risk of front-running looms large, in which case it may be optimal for the client to mitigate it by contacting only a single dealer. The risk of front-running is especially large, for example, if the dealers are likely to be initially long (which makes $\underline{\phi}$ large), while the client is ex ante likely to sell (i.e., ϕ_0 is small), and the client's realized trading need is indeed to sell. This is why claim *(iv)* of the proposition says that $q_{-\bar{s}} < 1$ is optimal when $\phi_0 \in (0, \underline{\phi})$. Symmetric intuition applies to claim *(v)*. In the other cases, the risk of front-running is small enough to endure in exchange for the countervailing benefits of an additional dealer. This is why claim *(iii)* says that $q_{-\bar{s}} = q_{\bar{s}} = 1$ is optimal when $\phi_0 \in [\underline{\phi}, \overline{\phi}]$, why claim *(iv)* says that $q_{\bar{s}} = 1$ is optimal when $\phi_0 \in (0, \underline{\phi})$, and why claim *(v)* says that $q_{-\bar{s}} = 1$ is optimal when $\phi_0 \in (\overline{\phi}, 1)$.

In recent years, electronic trading platforms have been introduced with the goal of reducing search costs for many asset classes that had traditionally traded over the counter. However, a puzzle is that many of these platforms have not been widely adopted (SIFMA Insights, 2019). Indeed, in many classic models of over-the-counter markets, traders would benefit from interventions that reduce or eliminate search costs. Based on this intuition, one would expect adoption of these electronic trading platforms. We provide a potential explanation for this puzzle. Our analysis demonstrates that it is not enough to simply eliminate search costs. So long as front-running remains a concern, clients might desire to restrict the number of dealers that they contact, and in particular might rationally avoid platforms that would expose their orders to a large number of potential counterparties. In that sense, the front-running effect that emerges from our model could be thought of as an endogenous search friction.

3.4.3 Examples

Asymmetric inventories. We illustrate with an example, which also provides a geometric interpretation of Proposition 4. Suppose $\psi = 0.85$ and $\rho = 1$. Then $\hat{c}_1$ and $\hat{c}_2(\phi)$ are as depicted in the first panel of Figure 1.[18] With this parametrization, $\hat{c}_2(\phi)$ is a decreasing function, and it in fact crosses $\hat{c}_1$ from above. The intuition is that when $\phi \approx 0$, the client is likely to be selling; at the same time, because $\psi = 0.85$, the dealers are likely to be long. Hence, if two dealers are contacted, the likely outcome is that the winning dealer will have to sell on the market, while the losing dealer will front-run, which raises the client's ultimate cost of procurement (relative to what it would have been if the losing dealer had not been contacted). On the other hand, when $\phi \approx 1$, the client is likely to be buying (and as before, the dealers are likely to be long). Hence, if two dealers are contacted, the likely outcome is that both would be able to internalize the client's order, both will bid aggressively for the order, and the client's cost of procurement will be small (relative to what it would have been if the losing dealer had not been contacted).

The second panel of Figure 1 depicts $C(\phi)$, which is the convexification of the lower envelope of $\{\hat{c}_1, \hat{c}_2(\phi)\}$. This second panel also depicts $\underline{\phi}$, which is defined to ensure that the line connecting $(0, \hat{c}_1)$ to $\big(\phi, \hat{c}_2(\phi)\big)$ is tangent to $c_2(\phi)$. Alternatively, this is the minimum value for which $C(\phi) = \hat{c}_2(\phi)$. We also have $\overline{\phi} = 1$ in this case, but we do not depict this in the figure because $\overline{\phi}$ plays no role what follows.

The third panel of the figure relates to case *(iii)* of Proposition 4. Here, we have $\phi_0 \in [\underline{\phi}, \overline{\phi}]$. The optimal RFQ policy always contacts two dealers and discloses no information about the client's order. Under this policy, dealers' beliefs therefore always coincide with the prior, so that the client's expected cost is $\hat{c}_2(\phi_0)$.

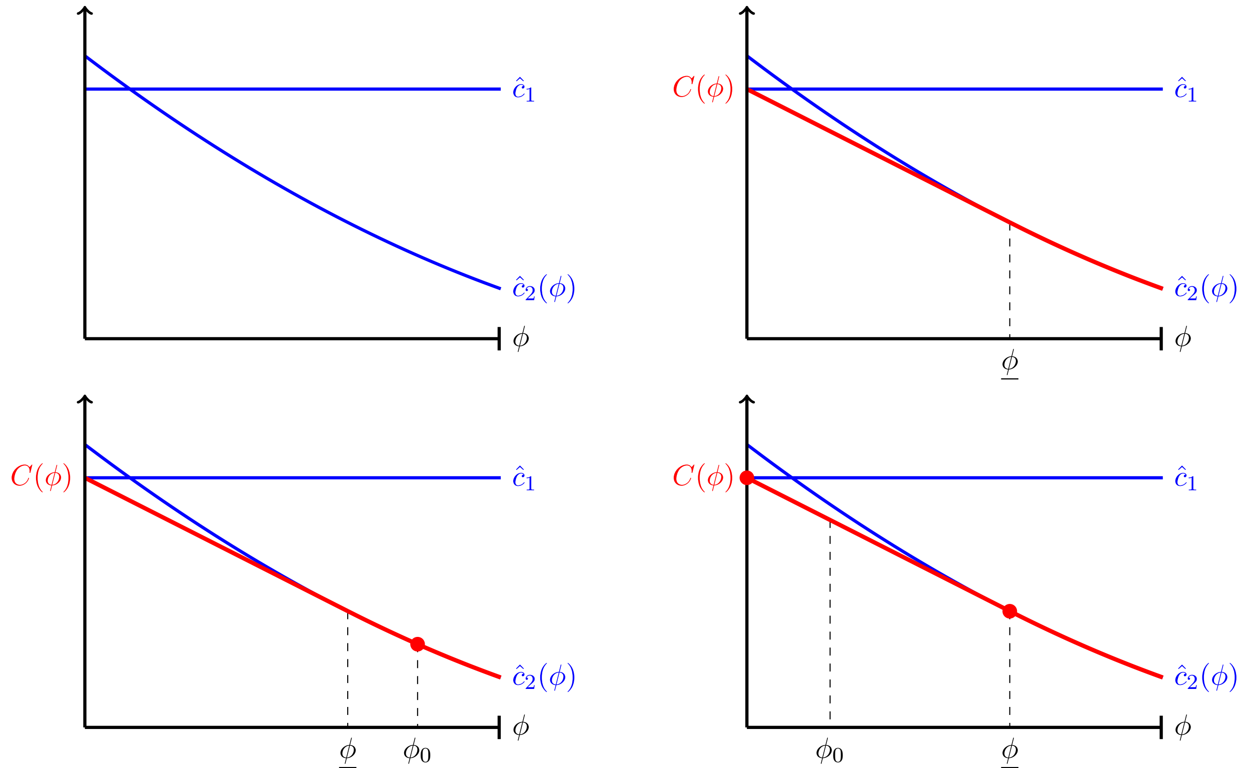
Finally, the fourth panel of the figure relates to case *(iv)* of Proposition 4. Here, we have $\phi_0 \in (0, \underline{\phi})$. The optimal RFQ policy always contacts two dealers when $s = \bar{s}$; and it mixes between one and two dealers when $s = -\bar{s}$. Hence, if one dealer is contacted, dealers believe $s = \bar{s}$ with probability 0. Moreover, the mixing that occurs when $s = -\bar{s}$ is designed to ensure that, conditional on two dealers being contacted, they are induced to believe that $s = \bar{s}$ with probability $\underline{\phi}$. No further information is disclosed beyond this. Under this policy, the client's expected procurement cost is therefore an appropriate convex combination of $\hat{c}_1$ and $\hat{c}_2(\underline{\phi})$, which is precisely what $C(\phi_0)$ captures.

Symmetric inventories. A potentially focal class of parametrizations consists of those cases in which dealers are equally likely to be long and short (i.e., $\psi = \frac{1}{2}$). In this case, $\hat{c}_2(\cdot)$ is minimized at $\phi = \frac{1}{2}$ and grows symmetrically in both directions to reach a maximum of $\hat{c}_2(0) = \hat{c}_2(1) = \frac{(15+\rho)\bar{s}^2}{32}$. It follows that regardless of ρ, $\hat{c}_2(\cdot)$ is everywhere less than $\hat{c}_1$, implying that $\underline{\phi} = 0$ and $\overline{\phi} = 1$, so that case *(iii)* of Proposition 4 always applies—that is, it is always optimal for the client to contact both dealers in these cases.

The intuition is that when dealers are equally likely to be long and short, it is always sufficiently likely that one of the dealers will be able to internalize the client's order—indeed, any one dealer will be able to internalize with probability $\frac{1}{2}$. It is therefore sufficiently *un*likely that contacting

[18] For $\bar{s} \in [0, 2]$, all cost quantities scale linearly with $\bar{s}^2$. Thus, because the figure does not specify units, it applies for all such $\bar{s}$.

Figure 1: Optimal RFQ Policies ($\psi = 0.85$ and $\rho = 1$)



an additional dealer would lead to front-running. With front-running rendered a sufficiently small concern, the client unambigously benefits from inducing additional competition for her order.

3.5 Testable implications

Our model makes several predictions that can be empirically tested. One set concerns information disclosure. If clients disclose more information about their order, then their procurement costs tend to increase on average. Moreover, this effect is stronger when more dealers are contacted. Following from this effect, another implication is that clients tend to avoid disclosing information when possible (e.g., by asking for a two-sided market). As mentioned, this prediction seems consistent with typical industry practice.

Another set of implications concerns the number of dealers contacted. Contacting more dealers tends to raise a client's procurement costs when dealers would have difficulty internalizing, but lowers procurement costs when dealers can more easily internalize. Following from this effect, another implication is that in situations where dealers tend to be long (e.g., for assets that are hard to short), clients will tend to contact more dealers when they want to buy than when they want to sell. Moreover, this effect is stronger when dealers become more likely to be long and for clients who are ex ante more likely to sell.

A final set of implications concerns on-market trading. Other dealers will initially tend to trade "with the wind" (i.e., in the same direction as the winning dealer), before reversing their direction to go "against the wind." Moreover, the amount of initial "with the wind" trading increases when more dealers are contacted and when more information is disclosed.

3.6 Policy implications

Our analysis so far has solved for how an unconstrained client would optimally behave. But in practice, clients are sometimes constrained by existing institutions and regulations; for these cases, our analysis has implications for when such constraints have potential to harm the client.

One application of our results is to regulations that require a minimum number of potential counterparties to be contacted in certain situations.[19] Our analysis suggests that there do exist circumstances in which concerns about front-running loom so large that a client would prefer to contact only a single dealer. In those cases, such mandates would be a binding constraint that could lead to suboptimal execution.

Another application is to the design of RFQ protocols, such as those used on SEFs. Many of these protocols require the client to reveal both the size and side of her desired transaction, effectively mandating an information policy of full disclosure.[20] Our analysis highlights that clients might benefit if these protocols were amended to permit more flexible information policies—in fact, our results imply that full disclosure is the *worst* information policy for the client.

A final application is to pre-trade transparency. To capture a population of heterogenous clients, consider a version of the model in which the parameter ϕ_0 is first drawn from a distribution F. Suppose further that the client is exogenously required to use the RFQ policy that always contacts two dealers and discloses no information; this simplifies the analysis by making the client non-strategic and shutting down the possibility that she could use the RFQ policy to signal her realized ϕ_0. A regime with pre-trade transparency might be captured by assuming that ϕ_0 is observed by the dealers in conjunction with the RFQ. In this regime, the average client procurement cost would be $\int \hat{c}_2(\phi)\, dF(\phi)$. A regime with pre-trade anonymity might be captured by assuming that ϕ_0 is never revealed. Letting $\phi_{avg} = \int \phi\, dF(\phi)$, the average client procurement cost would be then $\hat{c}_2(\phi_{avg})$, which is less, by the convexity of $\hat{c}_2(\cdot)$.[21] Our framework therefore contributes to the debate over regulations such as Dodd-Frank and MiFID II, parts of which were designed to enhance pre-trade transparency. To the extent that our model captures the mechanisms that are relevant for these markets, it suggests that these regulations might actually *increase* transaction costs for clients. From this perspective, pre-trade anonymity might be more desirable instead.

[19]Title VII of Dodd-Frank requires that for certain interest rate swaps and credit default swaps, all trades must be executed on SEFs. And for such swaps, the CFTC requires that at least three different market participants must be contacted for each RFQ.

[20]For example, this is the case for both the Bloomberg and Tradeweb SEFs, which are the top two in the index CDS market, according to data from the SEF Tracker, published by the Futures Industry Association.

[21]Thus, clients are better off on average under pre-trade anonymity. Of course, we are unlikely to have $\phi_{avg} = \arg\min \hat{c}_2(\phi)$, and so a subset of client types may be worse off. Note that if such client types could signal or disclose who they are, then we might expect the anonymous regime to unravel into the transparency regime (as in, e.g., Grossman, 1981; Milgrom, 1981). But the version of the model we are considering here shuts down the possibility for such signaling or disclosure.

4 Robustness

This section investigates the extent to which our results are robust to the assumption that the client has the ability to commit to an RFQ policy. We find that the result of Proposition 3 (on the optimality of no disclosure) does not hinge on the client's ability to commit: she would not want to deviate from her commitment not to disclose further information *even if she could*. In contrast, the result of Proposition 4 (on the optimal policy for determining the number of dealers to contact) does hinge on the commitment assumption. Motivated by this observation, we then investigate how that result would change if the client possessed only a weaker form of commitment power.

Preliminaries. To investigate these issues, it is useful to define interim analogues of $\hat{c}_1(\phi)$ and $\hat{c}_2(\phi)$. Suppose an RFQ contacts one dealer. Arguments analogous to those given before allow us to derive the client's minimum procurement cost conditional on her type: it is $\frac{3\bar{s}^2}{4}$, regardless of the client's realized type $s \in \{-\bar{s}, \bar{s}\}$ and regardless of the belief ϕ induced by the RFQ. Hence, we write $\hat{c}_{1,-\bar{s}} = \hat{c}_{1,\bar{s}} = \frac{3\bar{s}^2}{4}$. Note that these interim expressions agree with the ex ante expression that we had previously derived in that they satisfy $\hat{c}_1 = (1-\phi)\hat{c}_{1,-\bar{s}} + \phi\hat{c}_{1,\bar{s}}$.

Next, suppose an RFQ contacts two dealers and induces a belief ϕ. A derivation analogous to that in the proof of Lemma 2 allows us to characterize the client's expected procurement cost conditional on her type, $\hat{c}_{2,-\bar{s}}(\phi)$ and $\hat{c}_{2,\bar{s}}(\phi)$. These expressions similarly agree with the ex ante cost in that they satisfy $\hat{c}_2(\phi) = (1-\phi)\hat{c}_{2,-\bar{s}}(\phi) + \phi\hat{c}_{2,\bar{s}}(\phi)$. These expressions moreover imply the following result.

Lemma 5. *$\hat{c}'_{2,-\bar{s}}(\phi) \leq 0$ and $\hat{c}'_{2,\bar{s}}(\phi) \geq 0$ on the domain $\phi \in [0,1]$.*

Optimality of no disclosure. The optimality of no disclosure does not depend on the client's ability to commit. To argue this point, we first recall that information design does not matter when only one dealer is contacted. Hence, it suffices to focus on the case of $M = 2$. Given an arbitrary RFQ policy of the form described in Proposition 3 (i.e., entailing no disclosure), let ϕ_2 denote the posterior beliefs about the client's type induced by a realization of $M = 2$. Suppose the client's realized type is $s = \bar{s}$. If she follows through on her commitment not to disclose, then her expected procurement cost will be $\hat{c}_{2,\bar{s}}(\phi_2)$. Alternatively, suppose she were to deviate by verifiably revealing her type (as in, e.g., Grossman, 1981; Milgrom, 1981); in that case, her expected procurement cost would be $\hat{c}_{2,\bar{s}}(1)$. This is always greater, by Lemma 5. Symmetrically, the client would also not wish to deviate from her commitment not to disclose when her realized type is $s = -\bar{s}$.

Number of dealers to contact. In contrast, what may hinge on the client's ability to commit is her policy for determining the number of dealers to contact. To illustrate with an example, let us consider the parametrization corresponding to the fourth panel of Figure 1: $\psi = 0.85$, $\rho = 1$, and $\phi_0 = 0.2$. The optimal policy has the feature that under the realization $s = -\bar{s}$, the client randomizes between contacting one and two dealers—and in such a way that if two dealers are contacted then that induces a belief $\underline{\phi} = 0.632$. However, the client would not be indifferent between contacting one and two dealers at this interim stage, given those beliefs. Indeed, we can compute $\hat{c}_{1,-\bar{s}} = 0.75\bar{s}^2$ and $\hat{c}_{2,-\bar{s}}(\underline{\phi}) \approx 0.7289\bar{s}^2$. Thus, the client would wish to deviate from her commitment to sometimes contact only a single dealer when her realized type is $s = \bar{s}$.

If the client could not commit to randomization. We have previously attempted to justify our assumption of commitment power for the client by considerations of reputation and repeated interaction. However, one might reasonably question whether such considerations would support commitments to policies that entail randomization in the number of contacted dealers. And indeed, we have just shown that the client would sometimes be tempted to deviate from such commitments to randomize. It may therefore be natural to ask what the client would do if she had only a weaker form of commitment power.

For example, suppose the client could commit to a policy for the number of dealers to be contacted only as a deterministic function of her type. Indeed, since deviations from such a commitment are easy to detect (in contrast to deviations from commitments to particular randomization schemes), it is much more plausible that this weaker form of commitment power could be supported by repeated interaction.

By arguments similar to those given earlier, it again suffices to focus on RFQ policies of the form described in Proposition 3. But we now restrict ourselves to those policies where $q_{-\bar{s}}, q_{\bar{s}} \in \{0,1\}$. The optimal such policy can be characterized via the following counterpart to Proposition 4:

Definition 1′. Define $\underaccent{\tilde}{\phi}, \tilde{\phi} \in [0,1]$ as follows. If $\hat{c}_2(0) \le \hat{c}_1$, define $\underaccent{\tilde}{\phi} = 0$; if $\hat{c}_2(1) - \hat{c}_2'(1) \ge \hat{c}_1$, define $\underaccent{\tilde}{\phi} = 1$; otherwise, define it implicitly as the unique $\underaccent{\tilde}{\phi} \in (0,1)$ that solves $\phi\hat{c}_2(1) + (1-\phi)\hat{c}_1 = \hat{c}_2(\phi)$. If $\hat{c}_2(1) \le \hat{c}_1$, define $\tilde{\phi} = 1$; if $\hat{c}_2'(0) + \hat{c}_2(0) \ge \hat{c}_1$, define $\tilde{\phi} = 0$; otherwise, define it implicitly as the unique $\tilde{\phi} \in (0,1)$ that solves $\phi\hat{c}_1 + (1-\phi)\hat{c}_2(0) = \hat{c}_2(\phi)$.[22]

Proposition 4′. *Under this alternative version of the model, the following hold:*

(i) $\min\{\phi\hat{c}_2(1) + (1-\phi)\hat{c}_1, \phi\hat{c}_1 + (1-\phi)\hat{c}_2(0), \hat{c}_2(\phi_0)\}$ *is a lower bound on the client's cost of procurement.*

(ii) $\underaccent{\tilde}{\phi} \le \tilde{\phi}$.

(iii) If $\phi_0 \in [\underaccent{\tilde}{\phi}, \tilde{\phi}]$, *then this lower bound is achieved by the RFQ policy defined by* $q_{-\bar{s}} = q_{\bar{s}} = 1$.

(iv) If $\phi_0 \in (0, \underaccent{\tilde}{\phi})$, *then this lower bound is achieved by the RFQ policy defined by* $q_{-\bar{s}} = 0$ *and* $q_{\bar{s}} = 1$.

(v) If $\phi_0 \in (\tilde{\phi}, 1)$, *then this lower bound is achieved by the RFQ policy defined by* $q_{-\bar{s}} = 1$ *and* $q_{\bar{s}} = 0$.

The proof is similar to that of Proposition 4 and is therefore omitted. In addition, Appendix C illustrates the result by revisiting the example underlying Figure 1. We had previously shown that

[22] To see that this provides a well-defined definition for $\underaccent{\tilde}{\phi}$, suppose that $\hat{c}_2(0) > \hat{c}_1$ and $\hat{c}_2(1) - \hat{c}_2'(1) < \hat{c}_1$. Then $\phi\hat{c}_2(1) + (1-\phi)\hat{c}_1 - \hat{c}_2(\phi)$ is a concave function, which is negative at $\phi = 0$, zero at $\phi = 1$, and decreasing at $\phi = 1$. It therefore has a unique zero in the interval $(0,1)$.

concerns about front-running can act as a search friction, leading the client to limit the number of dealers that she contacts. What this result demonstrates is that that conclusion does not depend on the strong form of commitment power that we had assumed for the client. Even with the weaker version of commitment we are considering here, the client may still optimally contact only a single dealer.

5 Conclusion

The search for a suitable counterparty entails complex tradeoffs. Contacting more dealers increases the chance of finding a good match, for example, a dealer who can internalize parts of the trade. In addition, orchestrating competition reduces the market power that dealers may have. However, in settings with price impact, there is an opposing force in the form of information leakage. By contacting multiple dealers, the client reveals information to more parties, which may lead to front-running, and thus larger procurement costs. In this environment, it is not obvious how a client should procure fulfillment of her trade. Should she vary the number of dealers that she contacts for a quote—and if so, how? Should she provide information about her trade before contracting with a counterparty—and if so, what kind?

We develop a model to study this problem. A client, who is either a buyer or a seller of a security, contacts either one or two dealers for a quote and conducts a procurement auction. Dealers are initially either long or short. The client reveals her trading need to the winning dealer who may then trade on the market in two periods. The other dealer does not observe the client's type, yet he may also access the market, either to provide liquidity or to front-run.

For the client, we show that secrecy about her trade is always in her best interest. In equilibrium she therefore does not reveal whether she intends to buy or sell; or in other words, she requests quotes for a two-sided market. Finally, we show that there is merit in varying the number of dealers that the client contacts. Specifically, it is optimal to avoid contacting two dealers only if doing so would be especially likely to induce front-running (given her realized trading need and her prior over the dealers' initial inventories).

In addition to contributing a model of this search problem, our analysis also has implications for regulations affecting pre-trade transparency and for the design of request-for-quote protocols, such as those used on swap execution facilities.

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A Proofs

A.1 Proof of Lemma 1

To establish Lemma 1, we in fact prove the stronger result stated in Lemma A1.

Lemma A1. *In a subgame following an RFQ that contacts $M = 1$ dealer, the unique on-path equilibrium behavior is as follows. Dealer A bids*

$$(b^A_{-\bar{s}}, b^A_{\bar{s}}) = \begin{cases} \left(\frac{3\bar{s}^2}{4}, 0\right) & \text{if } (e^A, e^B) = (1,1) \\ \left(\frac{7\bar{s}^2}{16}, 0\right) & \text{if } (e^A, e^B) = (1,-1) \\ \left(0, \frac{7\bar{s}^2}{16}\right) & \text{if } (e^A, e^B) = (-1,1) \\ \left(0, \frac{3\bar{s}^2}{4}\right) & \text{if } (e^A, e^B) = (-1,-1) \end{cases}$$

If dealer A wins, the on-market trades are

$$(x^A_1, x^A_2, x^B_2) = \begin{cases} (0,0,0) & \text{if } (s, e^A, e^B) = (\bar{s}, 1, 1) \\ (-\frac{\bar{s}}{2}, -\frac{\bar{s}}{2}, 0) & \text{if } (s, e^A, e^B) = (-\bar{s}, 1, 1) \\ (0,0,0) & \text{if } (s, e^A, e^B) = (\bar{s}, 1, -1) \\ (-\frac{\bar{s}}{4}, -\frac{3\bar{s}}{4}, \frac{\bar{s}}{2}) & \text{if } (s, e^A, e^B) = (-\bar{s}, 1, -1) \\ (\frac{\bar{s}}{4}, \frac{3\bar{s}}{4}, -\frac{\bar{s}}{2}) & \text{if } (s, e^A, e^B) = (\bar{s}, -1, 1) \\ (0,0,0) & \text{if } (s, e^A, e^B) = (-\bar{s}, -1, 1) \\ (\frac{\bar{s}}{2}, \frac{\bar{s}}{2}, 0) & \text{if } (s, e^A, e^B) = (\bar{s}, -1, -1) \\ (0,0,0) & \text{if } (s, e^A, e^B) = (-\bar{s}, -1, -1) \end{cases}$$

Lemma 1 follows from Lemma A1 for reasons discussed in the main text. To ensure execution with probability one, the client's reserve must be at least the dealer's bid in the worst case. If the client wishes to sell, the worst case is $(e^A, e^B) = (1, 1)$, where $b^A_{-\bar{s}} = \frac{3\bar{s}^2}{4}$, according to Lemma A1. Symmetrically, if the client wishes to buy, the worst case is $(e^A, e^B) = (-1, -1)$, where $b^A_{\bar{s}} = \frac{3\bar{s}^2}{4}$. Therefore, it follows that the expected procurement cost resulting from an RFQ that contacts one dealer, induces a belief that ϕ is the probability of $s = \bar{s}$, and guarantees execution with probability one results is at least

$$\hat{c}_1 = \phi\frac{3\bar{s}^2}{4} + (1-\phi)\frac{3\bar{s}^2}{4} = \frac{3\bar{s}^2}{4},$$

as claimed by Lemma 1. Moreover, the RFQ that uses reserve prices $\bar{b} = (\frac{3\bar{s}^2}{4}, \frac{3\bar{s}^2}{4})$ both ensures execution with probability one and achieves the cost $\hat{c}_1$.

Proof of Lemma A1. Because both dealers observe the entire vector (e^A, e^B), the four possible realizations of that vector can be analyzed separately. Below, we analyze the cases of $(1, 1)$ and $(1, -1)$; the remaining cases can be handled symmetrically.

Case 1: $(e^A, e^B) = (1,1)$. Here is a full specification of an equilibrium for this case. Dealer A bids $\left(b^A_{-\bar{s}}, b^A_{\bar{s}}\right) = \left(\frac{3\bar{s}^2}{4}, 0\right)$. Henceforth, suppose that dealer A wins. If $s = \bar{s}$, dealer A sets $x_1^A = 0$ and

$$x_2^A = \begin{cases} -\frac{x_1^A}{3} & \text{if } 0 \leq x_1^A \leq \bar{s} \\ -\frac{x_1^A}{2} & \text{if } \max\{-\bar{s}, 2\bar{s} - 4\} \leq x_1^A < 0 \\ \bar{s} - 2 - x_1^A & \text{if } -\bar{s} \leq x_1^A \leq 2\bar{s} - 4 \end{cases} \tag{3}$$

If $s = -\bar{s}$, dealer A sets $x_1^A = -\frac{\bar{s}}{2}$ and

$$x_2^A = \begin{cases} -\bar{s} - x_1^A & \text{if } -\bar{s} \leq x_1^A \leq \bar{s} \end{cases} \tag{4}$$

Dealer B sets

$$x_2^B = \begin{cases} -\frac{x_1^A}{3} & \text{if } 0 \leq x_1^A \leq \bar{s} \\ 0 & \text{if } -\bar{s} \leq x_1^A < 0 \end{cases} \tag{5}$$

Dealers' beliefs prior to bidding are $\mu_0^A = \mu_0^B = \phi$. Dealer B's beliefs prior to second-period trading are

$$\mu_2^B = \begin{cases} 1 & \text{if } 0 \leq x_1^A \leq \bar{s} \\ 0 & \text{if } -\bar{s} \leq x_1^A < 0 \end{cases}$$

We claim that the specified strategies and beliefs satisfy the solution concept described in Section 3.1 and moreover that anything else also satisfying the solution concept must feature the same on-path behavior. The argument consists of three parts.

Part (i): We check the consistency of dealer B's beliefs. Given the specified strategy for dealer A, Bayes' rule requires only that

$$\mu_2^B = \begin{cases} 1 & \text{if } x_1^A = 0 \\ 0 & \text{if } x_1^A = -\frac{\bar{s}}{2} \end{cases}$$

This is indeed consistent with the specified beliefs.

Part (ii): Given the specified beliefs, we check that the solution concept uniquely pins down the specified strategies. We proceed by backward induction:

- *Period-2 reaction functions.* Dealer A's trading costs are $x_1^A x_1^A + (x_1^A + x_2^A + x_2^B)x_2^A$. For $s \in \{-\bar{s}, \bar{s}\}$, dealer A best responds with $x_2^A = \left[-\frac{x_1^A + x_2^B}{2}\right]_{s-2-x_1^A}^{s-x_1^A}$. Dealer B's trading costs are $(x_1^A + x_2^A + x_2^B)x_2^B$. Dealer B best responds with $x_2^B = \left[-\frac{x_1^A + x_2^A}{2}\right]_{-2}^{0}$.
- *Dealer B's period-2 action.* If $0 \leq x_1^A \leq \bar{s}$ so that $\mu_2^B = 1$, then x_2^B is pinned down by the intersection of $x_2^A = \left[-\frac{x_1^A + x_2^B}{2}\right]_{\bar{s}-2-x_1^A}^{\bar{s}-x_1^A}$ and $x_2^B = \left[-\frac{x_1^A + x_2^A}{2}\right]_{-2}^{0}$, so that we indeed have $x_2^B = -\frac{x_1^A}{3}$. If $-\bar{s} \leq x_1^A < 0$ so that $\mu_2^B = 0$, then x_2^B is pinned down by the intersection of $x_2^A =$

$\left[-\frac{x_1^A+x_2^B}{2}\right]_{-\bar{s}-2-x_1^A}^{-\bar{s}-x_1^A}$ and $x_2^B = \left[-\frac{x_1^A+x_2^A}{2}\right]_{-2}^{0}$, so that we indeed have $x_2^B = 0$. Together, these two cases verify (5).

- *Dealer A's period-2 action.* If $s = \bar{s}$, then x_2^A is pinned down by the intersection of $x_2^A = \left[-\frac{x_1^A+x_2^B}{2}\right]_{\bar{s}-2-x_1^A}^{\bar{s}-x_1^A}$ and (5), which verifies (3).

  If $s = -\bar{s}$, then x_2^A is pinned down by the intersection of $x_2^A = \left[-\frac{x_1^A+x_2^B}{2}\right]_{-\bar{s}-2-x_1^A}^{-\bar{s}-x_1^A}$ and (5), which verifies (4).

- *Dealer A's period-1 action.* Dealer A's trading costs are $x_1^A x_1^A + (x_1^A + x_2^A + x_2^B)x_2^A$. If $s = \bar{s}$, then we can plug in (3) and (5) to express dealer A's trading costs as a function of x_1^A:

$$\begin{cases} x_1^A x_1^A - (\frac{x_1^A}{3})^2 & \text{if } 0 \le x_1^A \le \bar{s} \\ x_1^A x_1^A - (\frac{x_1^A}{2})^2 & \text{if } \max\{s, 2\bar{s}-4\} \le x_1^A < 0 \\ x_1^A x_1^A + (\bar{s}-2)(\bar{s}-2-x_1^A) & \text{if } -\bar{s} \le x_1^A < 2\bar{s}-4 \end{cases}$$

  which is indeed minimized by $x_1^A = 0$.

  Alternatively, if $s = -\bar{s}$, then we can plug in (4) and (5) to express dealer A's trading costs as a function of x_1^A:

$$\begin{cases} x_1^A x_1^A + (-\bar{s} - \frac{x_1^A}{3})(-\bar{s} - x_1^A) & \text{if } 0 \le x_1^A \le \bar{s} \\ x_1^A x_1^A + (-\bar{s})(-\bar{s} - x_1^A) & \text{if } -\bar{s} \le x_1^A < 0 \end{cases}$$

  which is indeed minimized by $x_1^A = -\frac{\bar{s}}{2}$.

- *Dealer A's bid.* Plugging in the trading behavior derived above, we have the following. If $s = \bar{s}$, then dealer A's continuation utility is c if he wins and 0 if he loses. If $s = -\bar{s}$, then dealer A's continuation utility is $c - \frac{3}{4}\bar{s}^2$ if he wins and 0 if he loses. Based on the solution concept described in Section 3.1, dealer A must therefore bid $\left(b_{-\bar{s}}^A, b_{\bar{s}}^A\right) = \left(\frac{3}{4}\bar{s}^2, 0\right)$.

Part (iii): We check that the equilibrium specified above satisfies the restrictions on beliefs described in Section 3.1. And we also show that any equilibrium satisfying those conditions must feature on-path behavior coinciding with that of the equilibrium specified above. Let $\tilde{\mu}_2^B$ be candidate beliefs. One requirement is that $\tilde{\mu}_2^B$ must have the step-function structure described in the text.[23] We then partition the possible $\tilde{\mu}_2^B$ into three cases:

- Suppose, by way of contradiction, that we have an equilibrium with beliefs such that $\tilde{\mu}_2^B(0) = 0$. By the step-function structure of $\tilde{\mu}_2^B$ and because beliefs must be correct on path, this means that on path, dealer A sets $x_1^A > 0$ when $s = \bar{s}$. Arguments similar to those given above imply that we would subsequently have $(x_2^A, x_2^B) = (-\frac{x_1^A}{3}, -\frac{x_1^A}{3})$, leading to total trading

[23] That is, (i) for all $x \in [-\bar{s}, \bar{s}]$, $\tilde{\mu}_2^B(x) \in \{0, 1\}$, and (ii) for all $x' < x''$, $\tilde{\mu}_2^B(x') = 1 \implies \tilde{\mu}_2^B(x'') = 1$ and $\tilde{\mu}_2^B(x'') = 0 \implies \tilde{\mu}_2^B(x') = 0$.

costs for dealer A of $(x_1^A)^2 - (\frac{x_1^A}{3})^2 = \frac{8}{9}(x_1^A)^2$. On the other hand, suppose that dealer A deviated to set $x_1^A = 0$ when $s = \bar{s}$. Arguments similar to those given above imply that we would subsequently have $(x_2^A, x_2^B) = (0,0)$, leading to trading costs of 0. This constitutes a profitable deviation, contradicting the putative equilibrium.

- Suppose, by way of contradiction, that we have an equilibrium with beliefs such that $\tilde{\mu}_2^B(-\frac{\bar{s}}{2}) = 1$. By the step-function structure of $\tilde{\mu}_2^B$ and because beliefs must be correct on path, this means that on path, dealer A sets $x_1^A < -\frac{\bar{s}}{2}$ when $s = -\bar{s}$. Arguments similar to those given above imply that we would subsequently have $(x_2^A, x_2^B) = (-\bar{s} - x_1^A, 0)$, leading to trading costs for dealer A of $(x_1^A)^2 + (-\bar{s})(-\bar{s} - x_1^A) = \frac{3}{4}\bar{s}^2 + (\frac{\bar{s}}{2} + x_1^A)^2$. On the other hand, suppose that dealer A deviated to set $x_1^A = -\frac{\bar{s}}{2}$ when $s = \bar{s}$. Arguments similar to those given above imply that we would subsequently have $(x_2^A, x_2^B) = (-\frac{\bar{s}}{2}, 0)$, leading to trading costs of $\frac{3}{4}\bar{s}^2$. This constitutes a profitable deviation, contradicting the putative equilibrium.

- Finally, suppose we have an equilibrium with beliefs such that both $\tilde{\mu}_2^B(0) = 1$ and $\tilde{\mu}_2^B(-\frac{\bar{s}}{2}) = 0$. Using arguments similar to those given above, we can show that any such equilibrium induces the same on-path behavior as the equilibrium specified above. In particular, dealer A's equilibrium trading costs are as above: $C_*^A(\bar{s}) = 0$ and $C_*^A(-\bar{s}) = \frac{3}{4}\bar{s}^2$. We can also use arguments similar to those given above to compute

$$C^A(\bar{s}, x_1^A, 1) = \begin{cases} (x_1^A)^2 - (\frac{x_1^A}{3})^2 & \text{if } 0 \le x_1^A \le \bar{s} \\ (x_1^A)^2 - (\frac{x_1^A}{2})^2 & \text{if } 2\bar{s} - 4 \le x_1^A \le 0 \\ (x_1^A)^2 + (\bar{s} - 2)(\bar{s} - 2 - x_1^A) & \text{if } -\bar{s} \le x_1^A \le 2\bar{s} - 4 \end{cases}$$
$$C^A(\bar{s}, x_1^A, 0) = \begin{cases} (x_1^A)^2 + (-\bar{s})(-\bar{s} - x_1^A) & \text{if } -\bar{s} \le x_1^A \le \bar{s} \end{cases}$$

And so we conclude that for all $x_1^A \in [-\bar{s}, \bar{s}]$, $C^A(\bar{s}, x_1^A, 1) \ge 0 = C_*^A(\bar{s})$ and $C^A(-\bar{s}, x_1^A, 0) \ge \frac{3}{4}\bar{s}^2 = C_*^A(-\bar{s})$. Hence, the intuitive criterion does not rule out any such equilibrium (e.g., the equilibrium specified above).

<u>Case 2</u>: $(e^A, e^B) = (1, -1)$. Here is a full specification of an WPBE for this case. Dealer A bids $\left(b_{-\bar{s}}^A, b_{\bar{s}}^A\right) = \left(\frac{7\bar{s}^2}{16}, 0\right)$. Henceforth, suppose that dealer A wins. If $s = \bar{s}$, dealer A sets $x_1^A = 0$ and

$$x_2^A = \begin{cases} -\frac{x_1^A}{2} & \text{if } 0 \le x_1^A \le \bar{s} \\ -\frac{x_1^A}{3} & \text{if } \max\{\frac{3\bar{s}}{2} - 3, -\frac{\bar{s}}{6}\} \le x_1^A < 0 \\ \bar{s} - 2 - x_1^A & \text{if } -\frac{\bar{s}}{6} \le x_1^A < \frac{3\bar{s}}{2} - 3 \\ -\frac{x_1^A}{2} - \frac{\bar{s}}{4} & \text{if } \max\{-\bar{s}, \frac{5\bar{s}}{2} - 4\} \le x_1^A < -\frac{\bar{s}}{6} \\ \bar{s} - 2 - x_1^A & \text{if } -\bar{s} \le x_1^A < \min\{\frac{5\bar{s}}{2} - 4, -\frac{\bar{s}}{6}\} \end{cases} \tag{6}$$

If $s=-\bar{s}$, dealer A sets $x_1^A=-\frac{\bar{s}}{4}$ and

$$x_2^A=\begin{cases}-\bar{s}-x_1^A & \text{if } -\bar{s}\le x_1^A\le \bar{s}\end{cases} \tag{7}$$

Dealer B sets

$$x_2^B=\begin{cases}0 & \text{if } x_1^A\ge 0\\ -\frac{x_1^A}{3} & \text{if } \max\{\frac{3\bar{s}}{2}-3,-\frac{\bar{s}}{6}\}\le x_1^A<0\\ -\frac{\bar{s}}{2}+1 & \text{if } -\frac{\bar{s}}{6}\le x_1^A<\frac{3\bar{s}}{2}-3\\ \frac{\bar{s}}{2} & \text{if } -\bar{s}\le x_1^A<-\frac{\bar{s}}{6}\end{cases} \tag{8}$$

Dealers' beliefs prior to bidding are $\mu_0^A=\mu_0^B=\phi$. Dealer B's beliefs prior to second-period trading are

$$\mu_2^B=\begin{cases}1 & \text{if } -\frac{\bar{s}}{6}\le x_1^A\le \bar{s}\\ 0 & \text{if } -\bar{s}\le x_1^A<-\frac{\bar{s}}{6}\end{cases}$$

We claim that the specified strategies and beliefs satisfy the solution concept described in Section 3.1 and moreover that anything else also satisfying the solution concept must feature the same on-path behavior. The argument consists of three parts.

Part (i): We check the consistency of dealer B's beliefs. Given the specified strategy for dealer A, Bayes' rule requires only that

$$\mu_2^B=\begin{cases}1 & \text{if } x_1^A=0\\ 0 & \text{if } x_1^A=-\frac{\bar{s}}{4}\end{cases}$$

This is indeed consistent with the specified beliefs.

Part (ii): Given the specified beliefs, we check that the solution concept uniquely pins down the specified strategies. We proceed by backward induction:

- *Period-2 reaction functions.* Dealer A's trading costs are $x_1^Ax_1^A+(x_1^A+x_2^A+x_2^B)x_2^A$. For $s\in\{-\bar{s},\bar{s}\}$, dealer A best responds with $x_2^A=\left[-\frac{x_1^A+x_2^B}{2}\right]_{s-2-x_1^A}^{s-x_1^A}$. Dealer B's trading costs are $(x_1^A+x_2^A+x_2^B)x_2^B$. Dealer B best responds with $x_2^B=\left[-\frac{x_1^A+x_2^A}{2}\right]_0^2$.
- *Dealer B's period-2 action.* If $-\frac{\bar{s}}{6}\le x_1^A\le\bar{s}$ so that $\mu_2^B=1$, then x_2^B is pinned down by the intersection of $x_2^A=\left[-\frac{x_1^A+x_2^B}{2}\right]_{\bar{s}-2-x_1^A}^{\bar{s}-x_1^A}$ and $x_2^B=\left[-\frac{x_1^A+x_2^A}{2}\right]_0^2$, so that we indeed have

$$x_2^B=\begin{cases}0 & \text{if } x_1^A\ge 0\\ -\frac{x_1^A}{3} & \text{if } \max\{\frac{3\bar{s}}{2}-3,-\frac{\bar{s}}{6}\}\le x_1^A<0\\ -\frac{\bar{s}}{2}+1 & \text{if } -\frac{\bar{s}}{6}\le x_1^A<\frac{3\bar{s}}{2}-3\end{cases}$$

  If $-\bar{s}\le x_1^A<-\frac{\bar{s}}{6}$ so that $\mu_2^B=0$, then x_2^B is pinned down by the intersection of $x_2^A=$

$\left[-\frac{x_1^A+x_2^B}{2}\right]_{-\bar{s}-2-x_1^A}^{-\bar{s}-x_1^A}$ and $x_2^B = \left[-\frac{x_1^A+x_2^A}{2}\right]_0^2$, so that we indeed have $x_2^B = \frac{\bar{s}}{2}$. Together, these two cases verify (8).

- *Dealer A's period-2 action.* If $s = \bar{s}$, then x_2^A is pinned down by the intersection of $x_2^A = \left[-\frac{x_1^A+x_2^B}{2}\right]_{\bar{s}-2-x_1^A}^{\bar{s}-x_1^A}$ and (8), which verifies (6).

  If $s = -\bar{s}$, then x_2^A is pinned down by the intersection of $x_2^A = \left[-\frac{x_1^A+x_2^B}{2}\right]_{-\bar{s}-2-x_1^A}^{-\bar{s}-x_1^A}$ and (8), which verifies (7).

- *Dealer A's period-1 action.* Dealer A's trading costs are $x_1^A x_1^A + (x_1^A + x_2^A + x_2^B)x_2^A$. If $s = \bar{s}$, then we can plug in (6) and (8) to express dealer A's trading costs as a function of x_1^A:

$$\begin{cases} x_1^A x_1^A - (\frac{x_1^A}{2})^2 & \text{if } 0 \leq x_1^A \leq \bar{s} \\ x_1^A x_1^A - (\frac{x_1^A}{3})^2 & \text{if } \max\{\frac{3\bar{s}}{2} - 3, -\frac{\bar{s}}{6}\} \leq x_1^A < 0 \\ x_1^A x_1^A + (\frac{\bar{s}}{2} - 1)(\bar{s} - 2 - x_1^A) & \text{if } -\frac{\bar{s}}{6} \leq x_1^A < \frac{3\bar{s}}{2} - 3 \\ x_1^A x_1^A - (\frac{x_1^A}{2} + \frac{\bar{s}}{4})^2 & \text{if } \max\{-\bar{s}, \frac{5\bar{s}}{2} - 4\} \leq x_1^A < -\frac{\bar{s}}{6} \\ x_1^A x_1^A + (\frac{3\bar{s}}{2} - 2)(\bar{s} - 2 - x_1^A) & \text{if } -\bar{s} \leq x_1^A < \min\{\frac{5\bar{s}}{2} - 4, -\frac{\bar{s}}{6}\} \end{cases}$$

  which is indeed minimized by $x_1^A = 0$.

  Alternatively, if $s = -\bar{s}$, then we can plug in (7) and (8) to express dealer A's trading costs as a function of x_1^A:

$$\begin{cases} x_1^A x_1^A + (-\bar{s})(-\bar{s} - x_1^A) & \text{if } x_1^A \geq 0 \\ x_1^A x_1^A + (-\bar{s} - \frac{x_1^A}{3})(-\bar{s} - x_1^A) & \text{if } \max\{\frac{3\bar{s}}{2} - 3, -\frac{\bar{s}}{6}\} \leq x_1^A < 0 \\ x_1^A x_1^A + (-\frac{3\bar{s}}{2} + 1)(-\bar{s} - x_1^A) & \text{if } -\frac{\bar{s}}{6} \leq x_1^A < \frac{3\bar{s}}{2} - 3 \\ x_1^A x_1^A + (-\frac{\bar{s}}{2})(-\bar{s} - x_1^A) & \text{if } -\bar{s} \leq x_1^A < -\frac{\bar{s}}{6} \end{cases}$$

  which is indeed minimized by $x_1^A = -\frac{\bar{s}}{4}$.

- *Dealer A's bid.* Plugging in the trading behavior derived above, we have the following. If $s = \bar{s}$, then dealer A's continuation utility is c if he wins and 0 if he loses. If $s = -\bar{s}$, then dealer A's continuation utility is $c - \frac{7\bar{s}^2}{16}$ if he wins and 0 if he loses. Based on the solution concept described in Section 3.1, dealer A must therefore bid $\left(b_{-\bar{s}}^A, b_{\bar{s}}^A\right) = \left(\frac{7\bar{s}^2}{16}, 0\right)$.

Part (iii): We check that the equilibrium specified above satisfies the restrictions on beliefs described in Section 3.1. And we also show that any equilibrium satisfying those conditions must feature on-path behavior coinciding with that of the equilibrium specified above. Let $\tilde{\mu}_2^B$ be candidate beliefs. One requirement is that $\tilde{\mu}_2^B$ must have the step-function structure described in the text. We then partition the possible $\tilde{\mu}_2^B$ into three cases:

- Suppose, by way of contradiction, that we have an equilibrium with beliefs such that $\tilde{\mu}_2^B(-\frac{\bar{s}}{6}+\varepsilon) = 0$ for some $\varepsilon > 0$. By the step-function structure of $\tilde{\mu}_2^B$ and because beliefs must be correct on path, we can choose $\varepsilon > 0$ arbitrarily small (and in particular less than $\frac{2\bar{s}}{3}$). The step-function structure of $\tilde{\mu}_2^B$, together with the fact that beliefs must be correct on path, also implies that on path, dealer A sets $x_1^A > -\frac{\bar{s}}{6} + \varepsilon$ when $s = \bar{s}$. Arguments similar to those given above imply that we would subsequently have

$$(x_2^A, x_2^B) = \begin{cases} (-\frac{x_1^A}{2}, 0) & \text{if } 0 \leq x_1^A \leq \bar{s} \\ (-\frac{x_1^A}{3}, -\frac{x_1^A}{3}) & \text{if } -\frac{\bar{s}}{6} + \varepsilon < x_1^A \leq 0 \end{cases}$$

leading to total trading costs for dealer A of

$$\begin{cases} \frac{3}{4}(x_1^A)^2 & \text{if } 0 \leq x_1^A \leq \bar{s} \\ \frac{8}{9}(x_1^A)^2 & \text{if } -\frac{\bar{s}}{6} + \varepsilon < x_1^A \leq 0 \end{cases}$$

which are bounded below by 0. On the other hand, suppose that dealer A deviated to set $x_1^A = -\frac{\bar{s}}{6} + \varepsilon$ when $s = \bar{s}$. Arguments similar to those given above imply that we would subsequently have $(x_2^A, x_2^B) = (-\frac{\bar{s}}{6} - \frac{\varepsilon}{2}, \frac{\bar{s}}{2})$, leading to trading costs of $-\frac{\varepsilon}{4}(2\bar{s} - 3\varepsilon) < 0$. This constitutes a profitable deviation, contradicting the putative equilibrium.

- Suppose, by way of contradiction, that we have an equilibrium with beliefs such that $\tilde{\mu}_2^B(-\frac{\bar{s}}{4}) = 1$. By the step-function structure of $\tilde{\mu}_2^B$ and because beliefs must be correct on path, this means that on path, dealer A sets $x_1^A < -\frac{\bar{s}}{4}$ when $s = -\bar{s}$. Arguments similar to those given above imply that we would subsequently have $x_2^A = -\bar{s} - x_1^A$ and $x_2^B = \frac{\bar{s}}{2}$, leading to trading costs for dealer A of $C_*^A(-\bar{s}) = (x_1^A)^2 + \frac{\bar{s}}{2}(\bar{s} + x_1^A) = \frac{7}{16}\bar{s}^2 + (\frac{\bar{s}}{4} + x_1^A)^2$. On the other hand, suppose that dealer A deviated to set $x_1^A = -\frac{\bar{s}}{4}$ when $s = -\bar{s}$ and that—contrary to the putative beliefs—this were to induce a belief $\mu_2^B = 0$. Arguments similar to those given above imply that we would subsequently have $(x_2^A, x_2^B) = (-\frac{3\bar{s}}{4}, \frac{\bar{s}}{2})$, leading to trading costs of $C^A(-\bar{s}, -\frac{\bar{s}}{4}, 0) = \frac{7}{16}\bar{s}^2 < C_*^A(-\bar{s})$.

  Given that $\tilde{\mu}_2^B(-\frac{\bar{s}}{4}) = 1$, we can use arguments similar to those given above to show that on-path behavior coincides with that in the equilibrium specified above when $s = \bar{s}$. In particular, dealer A's equilibrium trading costs are as above: $C_*^A(\bar{s}) = 0$. On the other hand, suppose that dealer A deviated to set $x_1^A = -\frac{\bar{s}}{4}$ when $s = \bar{s}$ and that this were to induce a belief $\mu_2^B \in [0,1]$. Arguments similar to those given above imply that we would subsequently have $(x_2^A, x_2^B) = \left(\frac{(3\mu_2^B - 2)\bar{s}}{4(4-\mu_2^B)}, \frac{(8-7\mu_2^B)\bar{s}}{4(4-\mu_2^B)}\right)$, leading to trading costs of $C^A(\bar{s}, -\frac{\bar{s}}{4}, \mu_2^B) = \frac{(3-2\mu_2^B)(1+\mu_2^B)\bar{s}^2}{4(4-\mu_2^B)^2}$. This is minimized at $\mu_2^B = 0$, and hence $\min_{\mu_2^B \in [0,1]} C^A(\bar{s}, -\frac{\bar{s}}{4}, \mu_2^B) = \frac{3\bar{s}^2}{64} > C_*^A(\bar{s})$. The putative equilibrium therefore fails our intuitive criterion test.

- Finally, suppose we have an equilibrium with beliefs such that both $\tilde{\mu}_2^B(-\frac{\bar{s}}{6} + \varepsilon) = 1$ for all $\varepsilon > 0$ and $\tilde{\mu}_2^B(-\frac{\bar{s}}{4}) = 0$. Using arguments similar to those given above, we can show that any such equilibrium induces the same on-path behavior as the equilibrium specified above. In particular, dealer A's equilibrium trading costs are as above: $C_*^A(\bar{s}) = 0$ and $C_*^A(-\bar{s}) = \frac{7}{16}\bar{s}^2$. We can also use arguments similar to those given above to compute

$$C^A(\bar{s}, x_1^A, 1) = \begin{cases} (x_1^A)^2 - (\frac{x_1^A}{2})^2 & \text{if } 0 \le x_1^A \le \bar{s} \\ (x_1^A)^2 - (\frac{x_1^A}{3})^2 & \text{if } \frac{3\bar{s}}{2} - 3 \le x_1^A \le 0 \\ (x_1^A)^2 + (\frac{\bar{s}}{2} - 1)(\bar{s} - 2 - x_1^A) & \text{if } -\bar{s} \le x_1^A \le \frac{3\bar{s}}{2} - 3 \end{cases}$$

$$C^A(\bar{s}, x_1^A, 0) = \begin{cases} (x_1^A)^2 + (-\frac{\bar{s}}{2})(-\bar{s} - x_1^A) & \text{if } -\bar{s} \le x_1^A \le \bar{s} \end{cases}$$

And so we conclude that for all $x_1^A \in [-\bar{s}, \bar{s}]$, $C^A(\bar{s}, x_1^A, 1) \ge 0 = C_*^A(\bar{s})$ and $C^A(-\bar{s}, x_1^A, 0) \ge \frac{7}{16}\bar{s}^2 = C_*^A(-\bar{s})$. Hence, the intuitive criterion does not rule out any such equilibrium (e.g., the equilibrium specified above).

This completes the proof. □

A.2 Proof of Lemma 2

To establish Lemma 2, we in fact prove the stronger result stated in Lemma A2.

Lemma A2. *In a subgame following an RFQ that contacts $M = 2$ dealers, induces dealer beliefs ϕ, and entails reserve prices $\bar{b}_{-\bar{s}} \ge \frac{(2304-560\phi-157\phi^2)\bar{s}^2}{4(24-\phi)^2}$ and $\bar{b}_{\bar{s}} \ge \frac{(1587+874\phi-157\phi^2)\bar{s}^2}{4(23+\phi)^2}$, the unique on-path equilibrium behavior is as follows. Dealer A bids*

$$(b_{-\bar{s}}^A, b_{\bar{s}}^A) = \begin{cases} \left(\frac{(2304-560\phi-157\phi^2)\bar{s}^2}{4(24-\phi)^2}, -\frac{207(1-\phi)^2\bar{s}^2}{4(24-\phi)^2}\right) & \textit{if } (e^A, e^B) = (1,1) \\ \left(\frac{7\bar{s}^2}{16}, \frac{\bar{s}^2}{4}\right) & \textit{if } (e^A, e^B) = (1,-1) \\ \left(\frac{\bar{s}^2}{4}, \frac{7\bar{s}^2}{16}\right) & \textit{if } (e^A, e^B) = (-1,1) \\ \left(-\frac{207\phi^2\bar{s}^2}{4(23+\phi)^2}, \frac{(1587+874\phi-157\phi^2)\bar{s}^2}{4(23+\phi)^2}\right) & \textit{if } (e^A, e^B) = (-1,-1) \end{cases}$$

If dealer A wins, the on-market trades are

$$(x_1^A, x_2^A, x_1^B, x_2^B) = \begin{cases} \left(\frac{7(1-\phi)\bar{s}}{2(24-\phi)}, \frac{3(1-\phi)\bar{s}}{2(24-\phi)}, -\frac{8(1-\phi)\bar{s}}{24-\phi}, \frac{3(1-\phi)\bar{s}}{2(24-\phi)}\right) & \textit{if } (s, e^A, e^B) = (\bar{s}, 1, 1) \\ \left(-\frac{(16+7\phi)\bar{s}}{2(24-\phi)}, -\frac{(32-9\phi)\bar{s}}{2(24-\phi)}, -\frac{8(1-\phi)\bar{s}}{24-\phi}, \frac{8(1-\phi)\bar{s}}{24-\phi}\right) & \textit{if } (s, e^A, e^B) = (-\bar{s}, 1, 1) \\ (0,0,0,0) & \textit{if } (s, e^A, e^B) = (\bar{s}, 1, -1) \\ \left(-\frac{\bar{s}}{4}, -\frac{3\bar{s}}{4}, 0, \frac{\bar{s}}{2}\right) & \textit{if } (s, e^A, e^B) = (-\bar{s}, 1, -1) \\ \left(\frac{\bar{s}}{4}, \frac{3\bar{s}}{4}, 0, -\frac{\bar{s}}{2}\right) & \textit{if } (s, e^A, e^B) = (\bar{s}, -1, 1) \\ (0,0,0,0) & \textit{if } (s, e^A, e^B) = (-\bar{s}, -1, 1) \\ \left(\frac{(23-7\phi)\bar{s}}{2(23+\phi)}, \frac{(23+9\phi)\bar{s}}{2(23+\phi)}, \frac{8\phi\bar{s}}{23+\phi}, -\frac{8\phi\bar{s}}{23+\phi}\right) & \textit{if } (s, e^A, e^B) = (\bar{s}, -1, -1) \\ \left(-\frac{7\phi\bar{s}}{2(23+\phi)}, -\frac{3\phi\bar{s}}{2(23+\phi)}, \frac{8\phi\bar{s}}{23+\phi}, -\frac{3\phi\bar{s}}{2(23+\phi)}\right) & \textit{if } (s, e^A, e^B) = (-\bar{s}, -1, -1) \end{cases}$$

Dealer B's bids and the on-market trades if dealer B wins are specified symmetrically.

Lemma 2 follows from Lemma A2 for reasons discussed in the main text. For an RFQ that contacts two dealers to ensure execution with probability one, the client's reserve must be high enough to ensure that it never sets the price. Mathematically, this means $\bar{b}_{-\bar{s}} \geq \frac{(2304-560\phi-157\phi^2)\bar{s}^2}{4(24-\phi)^2}$ and $\bar{b}_{\bar{s}} \geq \frac{(1587+874\phi-157\phi^2)\bar{s}^2}{4(23+\phi)^2}$, where ϕ denotes the probability of $s = \bar{s}$ induced by the RFQ. These inequalities are in particular satisfied by $\bar{b} = (\bar{s}^2, \bar{s}^2)$.

It therefore follows from Lemma A2 that the expected cost of procurement achieved by an RFQ that contacts $M = 2$ dealers, induces a belief that ϕ is the probability of $s = \bar{s}$, and ensures execution with probability one can be computed as

$$\begin{aligned}\hat{c}_2(\phi) \equiv\; & \frac{(1-\phi)(2304-767\phi+50\phi^2)\bar{s}^2}{4(24-\phi)^2}\psi[1-(1-\psi)(1-\rho)] \\ & + \frac{\phi(1587+667\phi+50\phi^2)\bar{s}^2}{4(23+\phi)^2}(1-\psi)[1-\psi(1-\rho)] + \frac{7\bar{s}^2}{16}2\psi(1-\psi)(1-\rho).\end{aligned} \tag{9}$$

Indeed, with probability $\phi\psi[1-(1-\psi)(1-\rho)]$, we have $(s, e^A, e^B) = (\bar{s}, 1, 1)$, and the auction's clearing price is $-\frac{207(1-\phi)^2\bar{s}^2}{4(24-\phi)^2}$. With probability $(1-\phi)\psi[1-(1-\psi)(1-\rho)]$, we have $(s, e^A, e^B) = (-\bar{s}, 1, 1)$, and the auction's clearing price is $\frac{(2304-560\phi-157\phi^2)\bar{s}^2}{4(24-\phi)^2}$. With probability $\phi(1-\psi)[1-\psi(1-\rho)]$, we have $(s, e^A, e^B) = (\bar{s}, -1, -1)$, and the auction's clearing price is $\frac{(1587+874\phi-157\phi^2)\bar{s}^2}{4(23+\phi)^2}$. With probability $(1-\phi)(1-\psi)[1-\psi(1-\rho)]$, we have $(s, e^A, e^B) = (\bar{s}, -1, -1)$, and the auction's clearing price is $-\frac{207\phi^2\bar{s}^2}{4(23+\phi)^2}$. With the remaining probability $2\psi(1-\psi)(1-\rho)$, the auction's clearing price is $\max\{\frac{7\bar{s}^2}{16}, \frac{\bar{s}^2}{4}\} = \frac{7\bar{s}^2}{16}$. It follows that the expected procurement cost is as in (9).

The claims made by Lemma 2 about $\hat{c}_2(\phi)$ follow readily from (9). Indeed, we can see that it is differentiable and we can moreover compute

$$\hat{c}_2''(\phi) = \frac{529(624-95\phi)\bar{s}^2}{2(24-\phi)^4}\psi[1-(1-\psi)(1-\rho)] + \frac{529(529+95\phi)\bar{s}^2}{2(23+\phi)^4}(1-\psi)[1-\psi(1-\rho)],$$

which is positive on the domain $\phi \in [0, 1]$. We can also compute $\hat{c}_2(\frac{1}{2}) = \frac{1933\bar{s}^2}{4418}\psi[1-(1-\psi)(1-\rho)] + \frac{1933\bar{s}^2}{4418}(1-\psi)[1-\psi(1-\rho)] + \frac{7\bar{s}^2}{16}2\psi(1-\psi)(1-\rho) < \frac{3\bar{s}^2}{4} = \hat{c}_1$.

Proof of Lemma A2. Because both dealers observe the entire vector (e^A, e^B), the four possible realizations of that vector can be analyzed separately. Below, we analyze the cases of $(1, 1)$ and $(1, -1)$; the remaining cases can be handled symmetrically (i.e., by flipping signs and exchanging the roles of ϕ and $1-\phi$). Within each case, we focus on the event in which dealer A wins; the events in which dealer B wins can be handled symmetrically.

Case 1: $(e^A, e^B) = (1, 1)$. Here is a full specification of an WPBE for this case. Dealer A bids $\left(b^A_{-\bar{s}}, b^A_{\bar{s}}\right) = \left(\frac{(2304-560\phi-157\phi^2)\bar{s}^2}{4(24-\phi)^2}, -\frac{207(1-\phi)^2\bar{s}^2}{4(24-\phi)^2}\right)$. Henceforth, suppose that dealer A wins. If $s = \bar{s}$, dealer A sets $x_1^A = \frac{7(1-\phi)\bar{s}}{2(24-\phi)}$ and

$$x_2^A = \begin{cases} -\frac{x_1^A}{2} & \text{if } 0 \leq x_1^A \leq \bar{s} \text{ and } \frac{x_1^A}{2} \leq x_1^B \leq \bar{s} \\ -\frac{x_1^A + x_1^B}{3} & \text{if } 0 \leq x_1^A \leq \bar{s} \text{ and } -\bar{s} \leq x_1^B \leq \frac{x_1^A}{2} \\ -\frac{x_1^A}{2} & \text{if } \max\{-\bar{s}, 2\bar{s}-4\} \leq x_1^A < 0 \text{ and } -\bar{s} \leq x_1^B \leq \bar{s} \\ \bar{s} - 2 - x_1^A & \text{if } -\bar{s} \leq x_1^A < 2\bar{s} - 4 \text{ and } -\bar{s} \leq x_1^B \leq \bar{s} \end{cases} \tag{10}$$

If $s = -\bar{s}$, dealer A sets $x_1^A = -\frac{(16+7\phi)\bar{s}}{2(24-\phi)}$ and

$$x_2^A = \begin{cases} -\bar{s} - x_1^A & \text{if } -\bar{s} \leq x_1^A \leq \bar{s} \text{ and } -\bar{s} \leq x_1^B \leq \bar{s} \end{cases} \tag{11}$$

Dealer B sets $x_1^B = -\frac{8(1-\phi)\bar{s}}{24-\phi}$ and

$$x_2^B = \begin{cases} -x_1^B & \text{if } 0 \leq x_1^A \leq \bar{s} \text{ and } \frac{x_1^A}{2} \leq x_1^B \leq \bar{s} \\ -\frac{x_1^A + x_1^B}{3} & \text{if } 0 \leq x_1^A \leq \bar{s} \text{ and } -\bar{s} \leq x_1^B \leq \frac{x_1^A}{2} \\ -x_1^B & \text{if } -\bar{s} \leq x_1^A < 0 \text{ and } -\bar{s} \leq x_1^B \leq \bar{s} \end{cases} \tag{12}$$

Dealers' beliefs prior to bidding are $\mu_0^A = \mu_0^B = \phi$. Dealer B's beliefs prior to first-period trading are $\mu_1^B = \phi$. Dealer B's beliefs prior to second-period trading are

$$\mu_2^B = \begin{cases} 1 & \text{if } 0 \leq x_1^A \leq \bar{s} \\ 0 & \text{if } -\bar{s} \leq x_1^A < 0 \end{cases}$$

We claim that the specified strategies and beliefs satisfy the solution concept described in Section 3.1 and moreover that anything else also satisfying the solution concept must feature the same on-path behavior. The argument consists of three parts.

Part (i): We check the consistency of dealer B's beliefs. First, consider dealer B's beliefs (conditional on losing) at the point just after having observed the auction's outcome. Given symmetry of the specified bidding strategies and the fact that the auction's tie-breaking rule does not depend on the realized s, the auction's outcome is uninformative so that posterior beliefs must equal the prior. Thus, we indeed have $\mu_1^B = \phi$. Second, consider dealer B's beliefs (conditional on losing) at the point just after having observed the first trading period's outcome. Given the specified strategy for dealer A, Bayes' rule requires only that

$$\mu_2^B = \begin{cases} 1 & \text{if } x_1^A = \frac{7(1-\phi)\bar{s}}{2(24-\phi)} \\ 0 & \text{if } x_1^A = -\frac{(16+7\phi)\bar{s}}{2(24-\phi)} \end{cases}$$

This is indeed consistent with the specified beliefs.

Part (ii): Given the specified beliefs, we check that the solution concept uniquely pins down the specified strategies. We proceed by backward induction:

- *Period-2 reaction functions.* Dealer A's trading costs are $(x_1^A + x_1^B)x_1^A + (x_1^A + x_1^B + x_2^A + x_2^B)x_2^A$. For $s \in \{-\bar{s}, \bar{s}\}$, dealer A best responds with $x_2^A = \left[-\frac{x_1^A+x_1^B+x_2^B}{2}\right]_{s-2-x_1^A}^{s-x_1^A}$. Dealer B's trading costs are $(x_1^A+x_1^B)x_1^B+(x_1^A+x_1^B+x_2^A+x_2^B)x_2^B$. Dealer B best responds with $x_2^B = \left[-\frac{x_1^A+x_1^B+x_2^A}{2}\right]_{-2-x_1^B}^{-x_1^B}$.

- *Dealer B's period-2 action.* If $0 \le x_1^A \le \bar{s}$ so that $\mu_2^B = 1$, then x_2^B is pinned down by the intersection of $x_2^A = \left[-\frac{x_1^A+x_1^B+x_2^B}{2}\right]_{\bar{s}-2-x_1^A}^{\bar{s}-x_1^A}$ and $x_2^B = \left[-\frac{x_1^A+x_1^B+x_2^A}{2}\right]_{-2-x_1^B}^{-x_1^B}$, so that we indeed have:

$$x_2^B = \begin{cases} -x_1^B & \text{if } 0 \le x_1^A \le \bar{s} \text{ and } \frac{x_1^A}{2} \le x_1^B \le \bar{s} \\ -\frac{x_1^A+x_1^B}{3} & \text{if } 0 \le x_1^A \le \bar{s} \text{ and } -\bar{s} \le x_1^B \le \frac{x_1^A}{2} \end{cases}$$

  If $-\bar{s} \le x_1^A < 0$ so that $\mu_2^B = 0$, then x_2^B is pinned down by the intersection of $x_2^A = \left[-\frac{x_1^A+x_1^B+x_2^B}{2}\right]_{-\bar{s}-2-x_1^A}^{-\bar{s}-x_1^A}$ and $x_2^B = \left[-\frac{x_1^A+x_1^B+x_2^A}{2}\right]_{-2-x_1^B}^{-x_1^B}$, so that we indeed have:

$$x_2^B = \begin{cases} -x_1^B & \text{if } -\bar{s} \le x_1^A < 0 \text{ and } -\bar{s} \le x_1^B \le \bar{s} \end{cases}$$

  Together, these two cases verify (12).

- *Dealer A's period-2 action.* If $s = \bar{s}$, then x_2^A is pinned down by the intersection of $x_2^A = \left[-\frac{x_1^A+x_1^B+x_2^B}{2}\right]_{\bar{s}-2-x_1^A}^{\bar{s}-x_1^A}$ and (12), which verifies (10). If $s = -\bar{s}$, then x_2^A is pinned down by the intersection of $x_2^A = \left[-\frac{x_1^A+x_1^B+x_2^B}{2}\right]_{-\bar{s}-2-x_1^A}^{-\bar{s}-x_1^A}$ and (12), which verifies (11).

- *Period-1 actions.* Dealer A's trading costs are $(x_1^A + x_1^B)x_1^A + (x_1^A + x_1^B + x_2^A + x_2^B)x_2^A$. If $s = \bar{s}$, then we can plug in (10) and (12) to express dealer A's trading costs as a function of (x_1^A, x_1^B):

$$\begin{cases} (x_1^A + x_1^B)x_1^A - (\frac{x_1^A}{2})^2 & \text{if } 0 \le x_1^A \le \bar{s} \text{ and } \frac{x_1^A}{2} \le x_1^B \le \bar{s} \\ (x_1^A + x_1^B)x_1^A - (\frac{x_1^A+x_1^B}{3})^2 & \text{if } 0 \le x_1^A \le \bar{s} \text{ and } -\bar{s} \le x_1^B \le \frac{x_1^A}{2} \\ (x_1^A + x_1^B)x_1^A - (\frac{x_1^A}{2})^2 & \text{if } \max\{-\bar{s}, 2\bar{s}-4\} \le x_1^A < 0 \text{ and } -\bar{s} \le x_1^B \le \bar{s} \\ (x_1^A + x_1^B)x_1^A + (\bar{s}-2)(\bar{s}-2-x_1^A) & \text{if } -\bar{s} \le x_1^A < 2\bar{s}-4 \text{ and } -\bar{s} \le x_1^B \le \bar{s} \end{cases}$$

  Optimizing, we can express $x_1^A(\bar{s})$ in terms of x_1^B:

$$x_1^A(\bar{s}) = \begin{cases} -\frac{7x_1^B}{16} & \text{if } -\bar{s} \le x_1^B \le 0 \\ -\frac{2x_1^B}{3} & \text{if } 0 \le x_1^B \le \min\{-3\bar{s}+6, \bar{s}\} \\ \frac{\bar{s}}{2} - 1 - \frac{x_1^B}{2} & \text{if } -3\bar{s}+6 \le x_1^B \le \bar{s} \end{cases} \tag{13}$$

Alternatively, if $s = -\bar{s}$, then we can plug in (11) and (12) to express dealer A's trading costs as a function of (x_1^A, x_1^B):

$$\begin{cases} (x_1^A + x_1^B)x_1^A + (-\bar{s})(-\bar{s} - x_1^A) & \text{if } 0 \le x_1^A \le \bar{s} \text{ and } \frac{x_1^A}{2} \le x_1^B \le \bar{s} \\ (x_1^A + x_1^B)x_1^A + (-\bar{s} - \frac{x_1^A}{3} + \frac{2x_1^B}{3})(-\bar{s} - x_1^A) & \text{if } 0 \le x_1^A \le \bar{s} \text{ and } -\bar{s} \le x_1^B \le \frac{x_1^A}{2} \\ (x_1^A + x_1^B)x_1^A + (-\bar{s})(-\bar{s} - x_1^A) & \text{if } -\bar{s} \le x_1^A < 0 \text{ and } -\bar{s} \le x_1^B \le \bar{s} \end{cases}$$

Optimizing, we can express $x_1^A(-\bar{s})$ in terms of x_1^B:

$$x_1^A(-\bar{s}) = \begin{cases} -\frac{\bar{s}}{2} - \frac{x_1^B}{2} & \text{if } -\bar{s} \le x_1^B \le \bar{s} \end{cases} \tag{14}$$

The derivation of dealer B's equilibrium period-1 action consists of two parts. First, we now show that there is a unique equilibrium in which $x_1^B \in [-\bar{s}, 0]$. Suppose $s = \bar{s}$. If x_1^A is a best response to $\hat{x}_1^B \in [-\bar{s}, 0]$, then by (13), we have $x_1^A = -\frac{7\hat{x}_1^B}{16}$, so that $x_1^A \in [0, \frac{7\bar{s}}{16}]$. Dealer B's trading costs are $(x_1^A + x_1^B)x_1^B + (x_1^A + x_1^B + x_2^A + x_2^B)x_2^B$. We can then plug in (10) and (12) to express dealer B's trading costs as a function of $(x_1^B, \hat{x}_1^B)$:

$$C_{\bar{s}}^B(x_1^B, \hat{x}_1^B) = \begin{cases} (-\frac{7\hat{x}_1^B}{16} + x_1^B)x_1^B + (-\frac{7\hat{x}_1^B}{32})(-x_1^B) & \text{if } -\frac{7\hat{x}_1^B}{32} \le x_1^B \le \bar{s} \\ (-\frac{7\hat{x}_1^B}{16} + x_1^B)x_1^B - (-\frac{7\hat{x}_1^B}{48} + \frac{x_1^B}{3})^2 & \text{if } -\bar{s} \le x_1^B \le -\frac{7\hat{x}_1^B}{32} \end{cases}$$

Suppose $s = -\bar{s}$. If x_1^A is a best response to $\hat{x}_1^B \in [-\bar{s}, 0]$, then by (14), we have $x_1^A = -\frac{\bar{s}}{2} - \frac{\hat{x}_1^B}{2}$, so that $x_1^A \in [-\frac{\bar{s}}{2}, 0]$. We can then plug in (11) and (12) to express dealer B's trading costs as a function of $(x_1^B, \hat{x}_1^B)$:

$$C_{-\bar{s}}^B(x_1^B, \hat{x}_1^B) = \begin{cases} (-\frac{\bar{s}}{2} - \frac{\hat{x}_1^B}{2} + x_1^B)x_1^B + (-\bar{s})(-x_1^B) & \text{if } -\bar{s} \le x_1^B \le \bar{s} \end{cases}$$

Because ϕ represents the probability of $\bar{s}$, dealer B's expected trading costs as a function of $(x_1^B, \hat{x}_1^B)$ are

$$\phi C_{\bar{s}}^B(x_1^B, \hat{x}_1^B) + (1-\phi)C_{-\bar{s}}^B(x_1^B, \hat{x}_1^B),$$

which is minimized by $x_1^B = -\frac{9(1-\phi)\bar{s}}{4(9-\phi)} + \frac{(72-23\phi)\hat{x}_1^B}{32(9-\phi)}$. Equilibrium occurs when this optimal value of x_1^B coincides with $\hat{x}_1^B$, which indeed occurs at $x_1^B = -\frac{8(1-\phi)\bar{s}}{24-\phi}$.

Second, we show that there is no equilibrium in which $x_1^B > 0$. Suppose $s = \bar{s}$. Let $\hat{x}_1^B > 0$ and let $x_1^A(\hat{x}_1^B)$ be the best response from (13). We can then plug in (10) and (12) to express dealer B's trading costs against $x_1^A(\hat{x}_1^B)$ from setting $x_1^B = \hat{x}_1^B$:

$$C_{\bar{s}}^B(\hat{x}_1^B, \hat{x}_1^B) = \begin{cases} \frac{2}{3}(\hat{x}_1^B)^2 & \text{if } 0 \le \hat{x}_1^B \le \min\{-3\bar{s}+6, \bar{s}\} \\ \frac{(-\bar{s}+2+\hat{x}_1^B)\hat{x}_1^B}{2} & \text{if } -3\bar{s}+6 \le \hat{x}_1^B \le \bar{s} \end{cases}$$

Similarly, suppose $s = -\bar{s}$. Let $\hat{x}_1^B > 0$ and let $x_1^A(\hat{x}_1^B)$ be the best response from (14). We can then plug in (11) and (12) to express dealer B's trading costs against $x_1^A(\hat{x}_1^B)$ from setting $x_1^B = \hat{x}_1^B$:

$$C_{-\bar{s}}^B(\hat{x}_1^B, \hat{x}_1^B) = \begin{cases} \frac{(\bar{s}+\hat{x}_1^B)\hat{x}_1^B}{2} & \text{if } 0 \le \hat{x}_1^B \le \bar{s} \end{cases}$$

Because ϕ represents the probability of $\bar{s}$, dealer B's expected trading costs against $x_1^A(\hat{x}_1^B)$ from setting $x_1^B = \hat{x}_1^B$ would be $\phi C_{\bar{s}}^B(\hat{x}_1^B, \hat{x}_1^B) + (1-\phi)C_{-\bar{s}}^B(\hat{x}_1^B, \hat{x}_1^B)$, which by the above expressions is strictly positive. But this cannot correspond to an equilibrium because dealer B could do strictly better against such trading behavior by dealer A: setting $x_1^B = x_2^B = 0$ guarantees trading costs of zero.

In conclusion, we have derived dealer B's period-1 action as $x_1^B = -\frac{8(1-\phi)\bar{s}}{24-\phi}$. Plugging this into (13) and (14), we indeed obtain $x_1^A(\bar{s}) = \frac{7(1-\phi)\bar{s}}{2(24-\phi)}$ and $x_1^A(-\bar{s}) = -\frac{(16+7\phi)\bar{s}}{2(24-\phi)}$, respectively. Finally, given these period-1 actions, (10)–(12) imply that the following period-2 actions will occur on the equilibrium path

$$(x_2^A, x_2^B) = \begin{cases} \left(\frac{3(1-\phi)\bar{s}}{2(24-\phi)}, \frac{3(1-\phi)\bar{s}}{2(24-\phi)}\right) & \text{if } s = \bar{s} \\ \left(-\frac{(32-9\phi)\bar{s}}{2(24-\phi)}, \frac{8(1-\phi)\bar{s}}{24-\phi}\right) & \text{if } s = -\bar{s} \end{cases}$$

- *Bids.* Plugging in the trading behavior derived above, we have the following. If $s = \bar{s}$ and if dealer A wins, then dealer A's continuation utility is $c + \frac{18(1-\phi)^2\bar{s}^2}{(24-\phi)^2}$, and dealer B's continuation utility is $-\frac{135(1-\phi)^2\bar{s}^2}{4(24-\phi)^2}$. By symmetry, if dealer B wins, then dealer A's continuation utility would be $-\frac{135(1-\phi)^2\bar{s}^2}{4(24-\phi)^2}$.

  If $s = -\bar{s}$ and if dealer A wins, then dealer A's continuation utility is $c - \frac{(64+5\phi)(32-9\phi)\bar{s}^2}{4(24-\phi)^2}$, and dealer B's continuation utility is $\frac{4(1-\phi)(16+7\phi)\bar{s}^2}{(24-\phi)^2}$. By symmetry, if dealer B wins, then dealer A's continuation utility would be $\frac{4(1-\phi)(16+7\phi)\bar{s}^2}{(24-\phi)^2}$.

  Because the dealer sets a non-binding reserve, the event in which neither dealer wins is not relevant. Based on the solution concept described in Section 3.1, dealer A must therefore bid

$$(b_{-\bar{s}}^A, b_{\bar{s}}^A) = \left(\frac{(2304 - 560\phi - 157\phi^2)\bar{s}^2}{4(24-\phi)^2}, -\frac{207(1-\phi)^2\bar{s}^2}{4(24-\phi)^2}\right).$$

Part (iii): We check that the equilibrium specified above satisfies the restrictions on beliefs described in Section 3.1. And we also show that any equilibrium satisfying those conditions must feature on-path behavior coinciding with that of the equilibrium specified above. Let $\tilde{\mu}_2^B$ be candidate beliefs. One requirement is that $\tilde{\mu}_2^B$ must have the step-function structure described in the text. We then partition the possible $\tilde{\mu}_2^B$ into four cases:

- Suppose, by way of contradiction, that we have an equilibrium with beliefs such that $\tilde{\mu}_2^B(\frac{7(1-\phi)\bar{s}}{2(24-\phi)}) = 0$. By the step-function structure of $\tilde{\mu}_2^B$ and because beliefs must be correct on path, this means that on path, dealer A sets $x_1^A = x^* > \frac{7(1-\phi)\bar{s}}{2(24-\phi)}$ when $s = \bar{s}$. Arguments similar to those given above imply that we have $x_1^A = \frac{7\phi x^* - (9+7\phi)\bar{s}}{27+5\phi}$ when $s = -\bar{s}$ and $x_1^B = -\frac{14\phi x^* + 9(1-\phi)\bar{s}}{27+5\phi}$. And when $s = \bar{s}$, we subsequently have $x_2^A = x_2^B = -\frac{x^* + x_1^B}{3}$, leading to total trading costs for dealer A of

$$(x^* + x_1^B)x^* - \frac{1}{9}(x^* + x_1^B)^2, \tag{15}$$

evaluated at $x_1^B = -\frac{14\phi x^* + 9(1-\phi)\bar{s}}{27+5\phi}$. Now consider two cases:

- First, suppose there exists a $\varepsilon > 0$ such that $\tilde{\mu}_2^B(x^* - \varepsilon) = 1$. In that case, it follows from equation (15) that x^* can be a locally optimal choice for dealer A only if $x_1^A = -\frac{7x_1^B}{16}$. Given that $x_1^B = -\frac{14\phi x^* + 9(1-\phi)\bar{s}}{27+5\phi}$, this requires $x^* = \frac{7(1-\phi)\bar{s}}{2(24-\phi)}$, a contradiction.
- Second, suppose that for all $\varepsilon > 0$, $\tilde{\mu}_2^B(x^* - \varepsilon) = 0$. In that case, if dealer A deviates to $x_1^A = x^* - \varepsilon$ when $s = \bar{s}$, then we subsequently have $x_2^B = -x_1^B$ and $x_2^A = -\frac{x_1^A}{2}$, leading to total trading costs for dealer A that for small ε are well approximated by

$$(x^* + x_1^B)x^* - \frac{1}{4}(x^*)^2, \tag{16}$$

  evaluated at $x_1^B = -\frac{14\phi x^* + 9(1-\phi)\bar{s}}{27+5\phi}$. Comparing (16) to (15) and using $x_1^B = -\frac{14\phi x^* + 9(1-\phi)\bar{s}}{27+5\phi}$, we see that this is a profitable deviation if $x^* > \frac{6(1-\phi)\bar{s}}{45-\phi}$, which is implied by $x^* > \frac{7(1-\phi)\bar{s}}{2(24-\phi)}$.

- Thus, we know that $\tilde{\mu}_2^B(\frac{7(1-\phi)\bar{s}}{2(24-\phi)}) = 1$. Arguments similar to those given above imply that our only candidate equilibrium entails $x_1^A = \frac{7(1-\phi)\bar{s}}{2(24-\phi)}$ when $s = \bar{s}$, $x_1^A = -\frac{(16+7\phi)\bar{s}}{2(24-\phi)}$ when $s = -\bar{s}$, and $x_1^B = -\frac{8(1-\phi)\bar{s}}{24-\phi}$. And when $s = \bar{s}$, we subsequently have $x_2^A = x_2^B = -\frac{x_1^A + x_1^B}{3}$, leading to total trading costs for dealer A of

$$-\frac{18(1-\phi)^2\bar{s}^2}{(24-\phi)^2}. \tag{17}$$

  Now suppose, by way of contradiction, that we have an equilibrium with beliefs such that $\tilde{\mu}_2^B(\frac{2(1-\phi)(8-\sqrt{10})\bar{s}}{3(24-\phi)} + \varepsilon) = 0$ for some $\varepsilon > 0$. Suppose then that dealer A deviated to set $x_1^A = \frac{2(1-\phi)(8-\sqrt{10})\bar{s}}{3(24-\phi)} + \varepsilon$ when $s = \bar{s}$. Arguments similar to those given above imply that we would subsequently have $x_2^B = -\frac{x_1^B}{2}$ and $x_2^A = -\frac{x_1^A}{2}$, leading to total trading costs for dealer A of

$$(x_1^A + x_1^B)x_1^A - \frac{(x_1^A)^2}{4}. \tag{18}$$

  Comparing (18) to (17), we see that this is a profitable deviation for sufficiently small $\varepsilon > 0$.

- Suppose, by way of contradiction, that we have an equilibrium with beliefs such that $\tilde{\mu}_2^B(-\frac{(16+7\phi)\bar{s}}{2(24-\phi)}) = 1$. By the step-function structure of $\tilde{\mu}_2^B$ and because beliefs must be correct on path, this means that on path, dealer A sets $x_1^A = x^* < -\frac{(16+7\phi)\bar{s}}{2(24-\phi)}$ when $s = -\bar{s}$. Arguments similar to those given above imply that we have $x_1^A = \frac{7(1-\phi)(\bar{s}+x^*)}{32-9\phi}$ when $s = \bar{s}$ and $x_1^B = -\frac{16(1-\phi)(\bar{s}+x^*)}{32-9\phi}$. When $s = -\bar{s}$, we subsequently have $x_2^A = -\bar{s} - x_1^A$ and $x_2^B = -x_1^B$, leading to equilibrium trading costs for dealer A, denoted $C_*^A(-\bar{s})$, equal to

$$\left(x_1^A - \frac{16(1-\phi)(\bar{s}+x^*)}{32-9\phi}\right)x_1^A + \bar{s}(\bar{s}+x_1^A) \tag{19}$$

evaluated at $x_1^A = x^*$. Alternatively, when $s=\bar{s}$, we subsequently have $x_2^A = x_2^B = -\frac{x_1^A+x_1^B}{3}$. Plugging in, dealer A's equilibrium trading costs are

$$C_*^A(\bar{s}) = -\frac{72(1-\phi)^2(\bar{s}+x^*)^2}{(32-9\phi)^2}. \tag{20}$$

Now define $\hat{x} \equiv -\frac{(16+7\phi)\bar{s}-16(1-\phi)x^*}{2(32-9\phi)}$. Note that $\hat{x} > x^*$, which follows from $x^* < -\frac{(16+7\phi)\bar{s}}{2(24-\phi)}$. Suppose that dealer A deviated to set $x_1^A = \hat{x}$ when $s=-\bar{s}$ and that—perhaps contrary to the putative beliefs—this were to induce a belief $\mu_2^B = 0$. Arguments similar to those given above imply that we would subsequently have $x_2^A = -\bar{s} - x_1^A$ and $x_2^B = -x_1^B$, leading to trading costs for dealer A, denoted $C^A(-\bar{s},\hat{x},0)$, equal to (19) evaluated at $x_1^A = \hat{x}$. Note that this choice optimizes (19). On the other hand, because $x^* \neq \hat{x}$, it does not optimize (19). We therefore conclude $C^A(-\bar{s},\hat{x},0) < C_*^A(-\bar{s})$.

On the other hand, suppose that dealer A deviated to set $x_1^A = \hat{x}$ when $s=\bar{s}$ and that this were to induce a belief $\mu_2^B \in [0,1]$. Arguments similar to those given above imply that we would subsequently have

$$x_2^A = \frac{[16(1-\phi)x^* + (48-25\phi)\bar{s}]\mu_2^B}{2(32-9\phi)(4-\mu_2^B)} \qquad x_2^B = \frac{[(48-25\phi)\bar{s}+16(1-\phi)x^*](4-3\mu_2^B)}{2(32-9\phi)(4-\mu_2^B)},$$

leading to trading costs of

$$C^A\left(\bar{s},\hat{x},\mu_2^B\right) = \frac{2[(48-25\phi)\bar{s}+16(1-\phi)x^*]\left([2(16+7\phi)-(16+7\phi)\mu_2^B - 4(1-\phi)(\mu_2^B)^2]\bar{s} - 4(1-\phi)x^*[(\mu_2^B)^2 - 4\mu_2^B + 8]\right)}{(32-9\phi)^2(4-\mu_2^B)^2}$$

This is minimized at $\mu_2^B = 0$, and hence

$$\min_{\mu_2^B\in[0,1]} C^A\left(\bar{s},\hat{x},\mu_2^B\right) = \frac{[(48-25\phi)\bar{s}+16(1-\phi)x^*][(16+7\phi)\bar{s}-16(1-\phi)x^*]}{4(32-9\phi)^2}. \tag{21}$$

Comparing (21) to (20), it can be shown that $C_*^A(\bar{s}) < \min_{\mu_2^B\in[0,1]} C^A\left(\bar{s},\hat{x},\mu_2^B\right)$. The putative equilibrium therefore fails our intuitive criterion test.

- Finally, suppose we have an equilibrium with beliefs such that both $\tilde{\mu}_2^B(\frac{2(1-\phi)(8-\sqrt{10})\bar{s}}{3(24-\phi)}+\varepsilon) = 1$ for all $\varepsilon > 0$ and $\tilde{\mu}_2^B(-\frac{(16+7\phi)\bar{s}}{2(24-\phi)}) = 0$. Using arguments similar to those given above, we can show that any such equilibrium induces the same on-path behavior as the equilibrium specified above. In particular, dealer A's equilibrium trading costs are as above: $C_*^A(\bar{s}) = -\frac{18(1-\phi)^2\bar{s}^2}{24-\phi)^2}$ and $C_*^A(-\bar{s}) = \frac{(62+5\phi)(32-9\phi)\bar{s}^2}{4(24-\phi)^2}$. We can also use arguments similar to those given above to compute

$$C^A(\bar{s}, x_1^A, 1) = \begin{cases} \left(x_1^A - \frac{8(1-\phi)\bar{s}}{24-\phi}\right)(x_1^A) - \frac{1}{9}\left(x_1^A - \frac{8(1-\phi)\bar{s}}{24-\phi}\right)^2 & \text{if } -\frac{16(1-\phi)\bar{s}}{24-\phi} \leq x_1^A \leq \bar{s} \\ \left(x_1^A - \frac{8(1-\phi)\bar{s}}{24-\phi}\right)(x_1^A) - (\frac{x_1^A}{2})^2 & \text{if } 2\bar{s} - 4 \leq x_1^A \leq -\frac{16(1-\phi)\bar{s}}{24-\phi} \\ \left(x_1^A - \frac{8(1-\phi)\bar{s}}{24-\phi}\right)(x_1^A) + (\bar{s}-2)(\bar{s}-2-x_1^A) & \text{if } -\bar{s} \leq x_1^A \leq 2\bar{s} - 4 \end{cases}$$

$$C^A(\bar{s}, x_1^A, 0) = \begin{cases} \left(x_1^A - \frac{8(1-\phi)\bar{s}}{24-\phi}\right)(x_1^A) + \bar{s}(\bar{s} + x_1^A) & \text{if } -\bar{s} \leq x_1^A \leq \bar{s} \end{cases}$$

And so we conclude that for all $x_1^A \in [-\bar{s}, \bar{s}]$, $C^A(\bar{s}, x_1^A, 1) \geq C_*^A(\bar{s})$ and $C^A(-\bar{s}, x_1^A, 0) \geq C_*^A(-\bar{s})$. Hence, the intuitive criterion does not rule out any such equilibrium (e.g., the equilibrium specified above).

Case 2: $(e^A, e^B) = (1, -1)$. Here is a full specification of an WPBE for this case. Dealer A bids $\left(b_{-\bar{s}}^A, b_{\bar{s}}^A\right) = \left(\frac{7\bar{s}^2}{16}, \frac{\bar{s}^2}{4}\right)$. Henceforth, suppose that dealer A wins. If $s = \bar{s}$, dealer A sets $x_1^A = 0$ and

$$x_2^A = \begin{cases} -\frac{x_1^A + x_1^B}{3} & \text{if } \max\{\frac{3\bar{s}}{2} + \frac{x_1^B}{2} - 3, -\frac{\bar{s}}{6}\} \leq x_1^A \leq \bar{s} \text{ and } \frac{x_1^A}{2} \leq x_1^B \leq \bar{s} \\ \bar{s} - 2 - x_1^A & \text{if } -\frac{\bar{s}}{6} \leq x_1^A \leq \frac{3\bar{s}}{2} + \frac{x_1^B}{2} - 3 \text{ and } \max\{-\bar{s}, \bar{s}-2\} \leq x_1^B \leq \bar{s} \\ -\frac{x_1^A}{2} & \max\{-\frac{\bar{s}}{6}, 2\bar{s} - 4\} \leq x_1^A \leq \bar{s} \text{ and } -\bar{s} \leq x_1^B \leq \frac{x_1^A}{2} \\ \bar{s} - 2 - x_1^A & \text{if } -\frac{\bar{s}}{6} \leq x_1^A \leq 2\bar{s} - 4 \text{ and } -\bar{s} \leq x_1^B \leq \bar{s} - 2 \\ -\frac{\bar{s}}{4} - \frac{x_1^A}{2} - \frac{x_1^B}{4} & \text{if } \max\{-\bar{s}, \frac{5\bar{s} + x_1^B}{2} - 4\} \leq x_1^A < -\frac{\bar{s}}{6} \text{ and } -\bar{s} \leq x_1^B \leq \bar{s} \\ \bar{s} - 2 - x_1^A & \text{if } -\bar{s} \leq x_1^A < \min\{\frac{5\bar{s} + x_1^B}{2} - 4, -\frac{\bar{s}}{6}\} \text{ and } -\bar{s} \leq x_1^B \leq \bar{s} \end{cases} \tag{22}$$

If $s = -\bar{s}$, dealer A sets $x_1^A = -\frac{\bar{s}}{4}$ and

$$x_2^A = \begin{cases} -\bar{s} - x_1^A & \text{if } -\bar{s} \leq x_1^A \leq \bar{s} \text{ and } -\bar{s} \leq x_1^B \leq \bar{s} \end{cases} \tag{23}$$

Dealer B sets $x_1^B = 0$ and

$$x_2^B = \begin{cases} -\frac{x_1^A + x_1^B}{3} & \text{if } \max\{\frac{3\bar{s}}{2} + \frac{x_1^B}{2} - 3, -\frac{\bar{s}}{6}\} \leq x_1^A \leq \bar{s} \text{ and } \frac{x_1^A}{2} \leq x_1^B \leq \bar{s} \\ -\frac{\bar{s}}{2} + 1 - \frac{x_1^B}{2} & \text{if } -\frac{\bar{s}}{6} \leq x_1^A \leq \frac{3\bar{s}}{2} + \frac{x_1^B}{2} - 3 \text{ and } \max\{-\bar{s}, \bar{s}-2\} \leq x_1^B \leq \bar{s} \\ -x_1^B & \text{if } \max\{-\frac{\bar{s}}{6}, 2\bar{s} - 4\} \leq x_1^A \leq \bar{s} \text{ and } -\bar{s} \leq x_1^B \leq \frac{x_1^A}{2} \\ -x_1^B & \text{if } -\frac{\bar{s}}{6} \leq x_1^A \leq 2\bar{s} - 4 \text{ and } -\bar{s} \leq x_1^B \leq \bar{s} - 2 \\ \frac{\bar{s}}{2} - \frac{x_1^B}{2} & \text{if } -\bar{s} \leq x_1^A < -\frac{\bar{s}}{6} \text{ and } -\bar{s} \leq x_1^B \leq \bar{s} \end{cases} \tag{24}$$

Dealers' beliefs prior to bidding are $\mu_0^A = \mu_0^B = \phi$. Dealer B's beliefs prior to first-period trading are $\mu_1^B = 1$. Dealer B's beliefs prior to second-period trading are

$$\mu_2^B = \begin{cases} 1 & \text{if } -\frac{\bar{s}}{6} \leq x_1^A \leq \bar{s} \\ 0 & \text{if } -\bar{s} \leq x_1^A < -\frac{\bar{s}}{6} \end{cases}$$

We claim that the specified strategies and beliefs satisfy the solution concept described in Section 3.1 and moreover that anything else also satisfying the solution concept must feature the same on-path behavior. The argument consists of three parts.

Part (i): We check the consistency of dealer B's beliefs. First, consider dealer B's beliefs (conditional on losing) at the point just after having observed the auction's outcome. In this case, dealer A bids $\left(b^A_{-\bar{s}}, b^A_{\bar{s}}\right) = \left(\frac{7\bar{s}^2}{16}, \frac{\bar{s}^2}{4}\right)$, while dealer B bids $\left(b^B_{-\bar{s}}, b^B_{\bar{s}}\right) = \left(\frac{\bar{s}^2}{4}, \frac{7\bar{s}^2}{16}\right)$. It follows that dealer B loses iff $s = \bar{s}$. Thus, we indeed have $\mu^B_1 = 1$. Second, consider dealer B's beliefs (conditional on losing) at the point just after having observed the first trading period's outcome. Given the specified strategy for dealer A, Bayes' rule requires only that

$$\mu^B_2 = \begin{cases} 1 & \text{if } x^A_1 = 0 \\ 0 & \text{if } x^A_1 = -\frac{\bar{s}}{4} \end{cases}$$

This is indeed consistent with the specified beliefs.

Part (ii): Given the specified beliefs, we check that the solution concept uniquely pins down the specified strategies. We proceed by backward induction:

- *Period-2 reaction functions.* Dealer A's trading costs are $x^A_1 x^A_1 + (x^A_1 + x^A_2 + x^B_2)x^A_2$. For $s \in \{-\bar{s}, \bar{s}\}$, dealer A best responds with $x^A_2 = \left[-\frac{x^A_1 + x^B_1 + x^B_2}{2}\right]^{s - x^A_1}_{s-2-x^A_1}$. Dealer B's trading costs are $(x^A_1 + x^A_2 + x^B_2)x^B_2$. Dealer B best responds with $x^B_2 = \left[-\frac{x^A_1 + x^B_1 + x^A_2}{2}\right]^{2 - x^B_1}_{-x^B_1}$.

- *Dealer B's period-2 action.* If $-\frac{\bar{s}}{6} \le x^A_1 \le \bar{s}$ so that $\mu^B_2 = 1$, then x^B_2 is pinned down by the intersection of $x^A_2 = \left[-\frac{x^A_1 + x^B_1 + x^B_2}{2}\right]^{\bar{s} - x^A_1}_{\bar{s}-2-x^A_1}$ and $x^B_2 = \left[-\frac{x^A_1 + x^B_1 + x^A_2}{2}\right]^{2 - x^B_1}_{-x^B_1}$, so that we indeed have:

$$x^B_2 = \begin{cases} -\frac{x^A_1 + x^B_1}{3} & \text{if } \max\{\frac{3\bar{s}}{2} + \frac{x^B_1}{2} - 3, -\frac{\bar{s}}{6}\} \le x^A_1 \le \bar{s} \text{ and } \frac{x^A_1}{2} \le x^B_1 \le \bar{s} \\ -\frac{\bar{s}}{2} + 1 - \frac{x^B_1}{2} & \text{if } -\frac{\bar{s}}{6} \le x^A_1 \le \frac{3\bar{s}}{2} + \frac{x^B_1}{2} - 3 \text{ and } \max\{-\bar{s}, \bar{s} - 2\} \le x^B_1 \le \bar{s} \\ -x^B_1 & \text{if } \max\{-\frac{\bar{s}}{6}, 2\bar{s} - 4\} \le x^A_1 \le \bar{s} \text{ and } -\bar{s} \le x^B_1 \le \frac{x^A_1}{2} \\ -x^B_1 & \text{if } -\frac{\bar{s}}{6} \le x^A_1 \le 2\bar{s} - 4 \text{ and } -\bar{s} \le x^B_1 \le \bar{s} - 2 \end{cases}$$

  If $-\bar{s} \le x^A_1 < -\frac{\bar{s}}{6}$ so that $\mu^B_2 = 0$, then x^B_2 is pinned down by the intersection of $x^A_2 = \left[-\frac{x^A_1 + x^B_1 + x^B_2}{2}\right]^{-\bar{s} - x^A_1}_{-\bar{s}-2-x^A_1}$ and $x^B_2 = \left[-\frac{x^A_1 + x^B_1 + x^A_2}{2}\right]^{2 - x^B_1}_{-x^B_1}$, so that we indeed have:

$$x^B_2 = \left\{ \frac{\bar{s}}{2} - \frac{x^B_1}{2} \quad \text{if } -\bar{s} \le x^A_1 < -\frac{\bar{s}}{6} \text{ and } -\bar{s} \le x^B_1 \le \bar{s} \right.$$

  Together, these two cases verify (24).

- *Dealer A's period-2 action.* If $s = \bar{s}$, then x^A_2 is pinned down by the intersection of $x^A_2 = \left[-\frac{x^A_1 + x^B_1 + x^B_2}{2}\right]^{\bar{s} - x^A_1}_{\bar{s}-2-x^A_1}$ and (24), which verifies (22). If $s = -\bar{s}$, then x^A_2 is pinned down by the intersection of $x_2^A = \left[-\frac{x_1^A+x_1^B+x_2^B}{2}\right]_{-\bar{s}-2-x_1^A}^{-\bar{s}-x_1^A}$ and (24), which verifies (23).

- *Period-1 actions.* Dealer A's trading costs are $(x_1^A+x_1^B)x_1^A + (x_1^A+x_1^B+x_2^A+x_2^B)x_2^A$. If $s=\bar{s}$, then we can plug in (22) and (24) to express dealer A's trading costs as a function of (x_1^A, x_1^B):

$$\begin{cases} (x_1^A+x_1^B)x_1^A - (\frac{x_1^A+x_1^B}{3})^2 & \text{if } \max\{\frac{3\bar{s}}{2}+\frac{x_1^B}{2}-3, -\frac{\bar{s}}{6}\} \leq x_1^A \leq \bar{s} \text{ and } \frac{x_1^A}{2} \leq x_1^B \leq \bar{s} \\ (x_1^A+x_1^B)x_1^A + (\frac{\bar{s}}{2}-1+\frac{x_1^B}{2})(\bar{s}-2-x_1^A) & \text{if } -\frac{\bar{s}}{6} \leq x_1^A \leq \frac{3\bar{s}}{2}+\frac{x_1^B}{2}-3 \text{ and } \max\{-\bar{s}, \bar{s}-2\} \leq x_1^B \leq \bar{s} \\ (x_1^A+x_1^B)x_1^A - (\frac{x_1^A}{2})^2 & \max\{-\frac{\bar{s}}{6}, 2\bar{s}-4\} \leq x_1^A \leq \bar{s} \text{ and } -\bar{s} \leq x_1^B \leq \frac{x_1^A}{2} \\ (x_1^A+x_1^B)x_1^A + (\bar{s}-2)(\bar{s}-2-x_1^A) & \text{if } -\frac{\bar{s}}{6} \leq x_1^A \leq 2\bar{s}-4 \text{ and } -\bar{s} \leq x_1^B \leq \bar{s}-2 \\ (x_1^A+x_1^B)x_1^A - (\frac{\bar{s}}{4}+\frac{x_1^A}{2}+\frac{x_1^B}{4})^2 & \text{if } \max\{-\bar{s}, \frac{5\bar{s}+x_1^B}{2}-4\} \leq x_1^A < -\frac{\bar{s}}{6} \text{ and } -\bar{s} \leq x_1^B \leq \bar{s} \\ (x_1^A+x_1^B)x_1^A + (\frac{3\bar{s}}{2}-2+\frac{x_1^B}{2})(\bar{s}-2-x_1^A) & \text{if } -\bar{s} \leq x_1^A < \min\{\frac{5\bar{s}+x_1^B}{2}-4, -\frac{\bar{s}}{6}\} \text{ and } -\bar{s} \leq x_1^B \leq \bar{s} \end{cases}$$

Rather than to derive dealer A's function of best responses to x_1^B, we instead derive two auxiliary functions. First, we derive dealer A's best response among first-period trades satisfying $-\frac{\bar{s}}{6} \leq x_1^A \leq \bar{s}$:

$$x_1^A(x_1^B) = \begin{cases} -\frac{2x_1^B}{3} & \text{if } -\bar{s} \leq x_1^B \leq 0 \\ -\frac{7x_1^B}{16} & \text{if } 0 \leq x_1^B \leq \min\{\frac{8\bar{s}}{21}, \frac{8(2-\bar{s})}{5}\} \\ -\frac{\bar{s}}{6} & \text{if } \frac{8\bar{s}}{21} \leq x_1^B \leq -\frac{10\bar{s}}{3}+6 \\ \frac{3\bar{s}}{2}+\frac{x_1^B}{2}-3 & \text{if } \max\{\frac{8(2-\bar{s})}{5}, -\frac{10\bar{s}}{3}+6\} \leq x_1^B \leq \frac{5(2-\bar{s})}{3} \\ \frac{\bar{s}}{4}-\frac{1}{2}-\frac{x_1^B}{4} & \text{if } \frac{5(2-\bar{s})}{3} \leq x_1^B \leq \frac{5\bar{s}}{3}-2 \\ -\frac{\bar{s}}{6} & \text{if } \max\{-\frac{10\bar{s}}{3}+6, \frac{5\bar{s}}{3}-2\} \leq x_1^B \leq \bar{s} \end{cases} \tag{25}$$

Second, we derive dealer A's best response among first-period trades satisfying $x_1^A < -\frac{\bar{s}}{6}$. Because the domain is not compact, this function is not defined everywhere:

$$x_1^A(x_1^B) = \begin{cases} \text{undefined} & \text{if } -\bar{s} \leq x_1^B \leq \max\{\frac{2\bar{s}}{3}, \frac{11\bar{s}}{3}-4\} \\ \frac{\bar{s}}{6}-\frac{x_1^B}{2} & \text{if } \frac{2\bar{s}}{3} < x_1^B \leq -\frac{7\bar{s}}{3}+4 \\ \frac{3\bar{s}}{2}+\frac{x_1^B}{2}-3 & \text{if } \max\{-\frac{7\bar{s}}{3}+4, \frac{11\bar{s}}{3}-4\} < x_1^B \leq \bar{s} \end{cases} \tag{26}$$

Dealer B's trading costs are $(x_1^A+x_1^B)x_1^B + (x_1^A+x_1^B+x_2^A+x_2^B)x_2^B$. If $s=\bar{s}$, then we can plug in (22) and (24) to express dealer B's trading costs as a function of (x_1^A, x_1^B):

$$\begin{cases} (x_1^A + x_1^B)x_1^A - (\frac{x_1^A + x_1^B}{3})^2 & \text{if } \max\{\frac{3\bar{s}}{2} + \frac{x_1^B}{2} - 3, -\frac{\bar{s}}{6}\} \le x_1^A \le \bar{s} \text{ and } \frac{x_1^A}{2} \le x_1^B \le \bar{s} \\ (x_1^A + x_1^B)x_1^B - (\frac{\bar{s}}{2} - 1 + \frac{x_1^B}{2})^2 & \text{if } -\frac{\bar{s}}{6} \le x_1^A \le \frac{3\bar{s}}{2} + \frac{x_1^B}{2} - 3 \text{ and } \max\{-\bar{s}, \bar{s} - 2\} \le x_1^B \le \bar{s} \\ (x_1^A + x_1^B)x_1^B + (\frac{x_1^A}{2})(-x_1^B) & \max\{-\frac{\bar{s}}{6}, 2\bar{s} - 4\} \le x_1^A \le \bar{s} \text{ and } -\bar{s} \le x_1^B \le \frac{x_1^A}{2} \\ (x_1^A + x_1^B)x_1^B + (\bar{s} - 2)(-x_1^B) & \text{if } -\frac{\bar{s}}{6} \le x_1^A \le 2\bar{s} - 4 \text{ and } -\bar{s} \le x_1^B \le \bar{s} - 2 \\ (x_1^A + x_1^B)x_1^B + (\frac{\bar{s}}{4} + \frac{x_1^A}{2} + \frac{x_1^B}{4})(\frac{\bar{s}}{2} - \frac{x_1^B}{2}) & \text{if } \max\{-\bar{s}, \frac{5\bar{s} + x_1^B}{2} - 4\} \le x_1^A < -\frac{\bar{s}}{6} \text{ and } -\bar{s} \le x_1^B \le \bar{s} \\ (x_1^A + x_1^B)x_1^B + (\frac{3\bar{s}}{2} - 2 + \frac{x_1^B}{2})(\frac{\bar{s}}{2} - \frac{x_1^B}{2}) & \text{if } -\bar{s} \le x_1^A < \min\{\frac{5\bar{s} + x_1^B}{2} - 4, -\frac{\bar{s}}{6}\} \text{ and } -\bar{s} \le x_1^B \le \bar{s} \end{cases}$$

Because dealer B attaches probability one to $s = \bar{s}$ (i.e., $\mu_1^B = 1$), dealer B selects x_1^B to minimize this objective. Optimizing, we can express x_1^B in terms of x_1^A:

$$x_1^B(x_1^A) = \begin{cases} \frac{\bar{s}-2}{3} - \frac{2x_1^A}{3} & \text{if } -\bar{s} \le x_1^A \le \max\{-\frac{\bar{s}}{6}, \frac{16(\bar{s}-2)}{13}\} \\ -\frac{7x_1^A}{16} & \text{if } \max\{-\frac{\bar{s}}{6}, \frac{16(\bar{s}-2)}{13}\} \le x_1^A \le 0 \\ -\frac{x_1^A}{4} & \text{if } 0 \le x_1^A \le \bar{s} \end{cases} \tag{27}$$

To determine equilibrium period-1 actions, we argue as follows. First, observe that (26) and (27) do not intersect. This implies that there is no equilibrium in which $x_1^A \in [-\bar{s}, -\frac{\bar{s}}{6})$ when $s = \bar{s}$. Second, observe that (25) and (27) have a unique intersection at $(x_1^A, x_1^B) = (0, 0)$. This is the unique candidate for an equilibrium involving a choice of $x_1^A \in [-\frac{\bar{s}}{6}, \bar{s}]$ when $s = \bar{s}$. To show that this is in fact an equilibrium, we simply need to additionally verify that no choice of $x_1^A \in [-\bar{s}, -\frac{\bar{s}}{6})$ yields smaller trading costs for dealer A's than $x_1^A = 0$ when $s = \bar{s}$ and $x_1^B = 0$, which is easily shown.[24]

Having pinned down $x_1^B = 0$ and that $x_1^A = 0$ if $s = \bar{s}$, the final step is to derive what x_1^A would be if $s = -\bar{s}$. Using $x_1^B = 0$, as derived above, we can plug in (23) and (24) to express dealer A's trading costs as a function of x_1^A:

$$x_2^B = \begin{cases} x_1^A x_1^A + (-\bar{s})(-\bar{s} - x_1^A) & \text{if } 0 \le x_1^A \le \bar{s} \\ x_1^A x_1^A + (-\bar{s} - \frac{x_1^A}{3})(-\bar{s} - x_1^A) & \text{if } \max\{\frac{3\bar{s}}{2} - 3, -\frac{\bar{s}}{6}\} \le x_1^A \le 0 \\ x_1^A x_1^A + (-\frac{3\bar{s}}{2} + 1)(-\bar{s} - x_1^A) & \text{if } -\frac{\bar{s}}{6} \le x_1^A \le \frac{3\bar{s}}{2} - 3 \\ x_1^A x_1^A + (-\frac{\bar{s}}{2})(-\bar{s} - x_1^A) & \text{if } -\bar{s} \le x_1^A < -\frac{\bar{s}}{6} \end{cases}$$

which is indeed minimized by $x_1^A = -\frac{\bar{s}}{4}$.

[24] Plugging $x_1^B = 0$ into the previously-derived expression for dealer A's trading costs when $s = \bar{s}$, we obtain that a choice of $x_1^A = 0$ yields trading costs of zero and that choices of $x_1^A \in [-\bar{s}, -\frac{\bar{s}}{6})$ yield the strictly positive trading costs

$$\begin{cases} \frac{1}{16}(2x_1^A - \bar{s})(6x_1^A + \bar{s}) & \text{if } \max\{-\bar{s}, \frac{5\bar{s}}{2} - 4\} \le x_1^A < -\frac{\bar{s}}{6} \\ \frac{1}{16}(2x_1^A - \bar{s})(6x_1^A + \bar{s}) + \frac{1}{16}(5\bar{s} - 2x_1^A - 8)^2 & \text{if } -\bar{s} \le x_1^A < \min\{\frac{5\bar{s}}{2} - 4, -\frac{\bar{s}}{6}\} \end{cases}$$

- *Bids.* Plugging in the trading behavior derived above, we have the following. If $s = \bar{s}$ and if dealer A wins, then dealer A's continuation utility is c, and dealer B's continuation utility is 0. If $s = -\bar{s}$ and if dealer A wins, then dealer A's continuation utility is $c - \frac{7\bar{s}^2}{16}$, and dealer B's continuation utility is $\frac{\bar{s}^2}{4}$.

  By symmetry, if $s = \bar{s}$ and if dealer B wins, then dealer A's continuation utility is $\frac{\bar{s}^2}{4}$. If $s = -\bar{s}$ and if dealer B wins, then dealer A's continuation utility is 0. Because the dealer sets a non-binding reserve, the event in which neither dealer wins is not relevant. Based on the solution concept described in Section 3.1, dealer A must therefore bid

$$(b^A_{-\bar{s}}, b^A_{\bar{s}}) = \left(\frac{7\bar{s}^2}{16}, \frac{\bar{s}^2}{4}\right).$$

Part (iii): We check that the equilibrium specified above satisfies the restrictions on beliefs described in Section 3.1. And we also show that any equilibrium satisfying those conditions must feature on-path behavior coinciding with that of the equilibrium specified above. Let $\tilde{\mu}^B_2$ be candidate beliefs. One requirement is that $\tilde{\mu}^B_2$ must have the step-function structure described in the text. We then partition the possible $\tilde{\mu}^B_2$ into four cases:

- Suppose, by way of contradiction, that we have an equilibrium with beliefs such that $\tilde{\mu}^B_2(0) = 0$. By the step-function structure of $\tilde{\mu}^B_2$ and because beliefs must be correct on path, this means that on path, dealer A sets $x^A_1 = x^* > 0$ when $s = \bar{s}$. Arguments similar to those given above imply that we have $x^B_1 = -\frac{x^*}{4}$, and subsequently, $x^A_2 = -\frac{x^*}{2}$ and $x^B_2 = -x^B_1$, leading to total trading costs for dealer A of

$$(x^* + x^B_1)x^* - \frac{1}{4}(x^*)^2, \tag{28}$$

  evaluated at $x^B_1 = -\frac{x^*}{4}$. Now consider two cases:

  - First, suppose there exists a $\varepsilon > 0$ such that $\tilde{\mu}^B_2(x^* - \varepsilon) = 1$. In that case, it follows from equation (28) that x^* can be a locally optimal choice for dealer A only if $x^A_1 = -\frac{2x^B_1}{3}$. Given that $x^B_1 = -\frac{x^*}{4}$, this requires $x^* = 0$, a contradiction.
  - Second, suppose that for all $\varepsilon > 0$, $\tilde{\mu}^B_2(x^* - \varepsilon) = 0$. In that case, if dealer A deviates to $x^A_1 = x^* - \varepsilon$ when $s = \bar{s}$, then we subsequently have $x^B_2 = \frac{\bar{s}}{2} - \frac{x^B_1}{2}$ and $x^A_2 = -\frac{\bar{s} + 2x^A_1 + x^B_1}{4}$, leading to total trading costs for dealer A that for small ε are well approximated by

$$(x^* + x^B_1)x^* - \left(\frac{\bar{s}}{4} + \frac{7x^*}{16}\right)^2, \tag{29}$$

    evaluated at $x^B_1 = -\frac{x^*}{4}$. Comparing (29) to (28) and using $x^* > 0$, we see that this is a profitable deviation.

- Thus, we know that $\tilde{\mu}^B_2(0) = 1$. Arguments similar to those given above imply that our only candidate equilibrium entails that when $s = \bar{s}$, $x^A_1 = 0$, $x^B_1 = 0$, and subsequently,

$x_2^A = x_2^B = 0$, leading to total trading costs for dealer A of 0. Now suppose, by way of contradiction, that we have an equilibrium with beliefs such that $\tilde{\mu}_2^B(-\frac{\bar{s}}{6}+\varepsilon) = 0$ for some $\varepsilon > 0$. By the step-function structure of $\tilde{\mu}_2^B$ and because beliefs must be correct on path, we can choose $\varepsilon > 0$ arbitrarily small (and in particular less than $\frac{2\bar{s}}{3}$). Suppose then that dealer A deviated to set $x_1^A = -\frac{\bar{s}}{6}+\varepsilon$ when $s = \bar{s}$. Arguments similar to those given above imply that we would subsequently have $x_2^B = \frac{\bar{s}}{2}$ and $x_2^A = -\frac{\bar{s}}{4} - \frac{x_1^A}{2}$, leading to total trading costs for dealer A of $-\frac{\varepsilon}{4}(2\bar{s} - 3\varepsilon)$, which is a profitable deviation for sufficiently small $\varepsilon > 0$.

- Suppose, by way of contradiction, that we have an equilibrium with beliefs such that $\tilde{\mu}_2^B(-\frac{\bar{s}}{4}) = 1$. The same arguments used in the proof of Lemma 1 can be used to show that the putative equilibrium fails our intuitive criterion test.

- Finally, suppose we have an equilibrium with beliefs such that both $\tilde{\mu}_2^B(-\frac{\bar{s}}{6}+\varepsilon) = 1$ for all $\varepsilon > 0$ and $\tilde{\mu}_2^B(-\frac{\bar{s}}{4}) = 0$. The same arguments used in the proof of Lemma 1 can be used to show that the intuitive criterion does not rule out any such equilibrium (e.g., the equilibrium specified above).

This completes the proof. □

A.3 Proof of Proposition 3

Proof. Begin with an arbitrary RFQ policy that contacts two dealers with probabilities $(q_{s'})_{s' \in \{-\bar{s}, \bar{s}\}}$ and one dealer with the complementary probabilities $(1 - q_{s'})_{s' \in \{-\bar{s}, \bar{s}\}}$. It suffices to show that the following RFQ policy does no worse: (*i*) define Σ as the singleton $\{\sigma_0\}$; (*ii*) for $s' \in \{-\bar{s}, \bar{s}\}$, define the distribution $\pi_{s'}$ to attach probability $q_{s'}$ to $\big(\sigma_0, 2, (\bar{s}^2, \bar{s}^2)\big)$ and complementary probability $1 - q_{s'}$ to $\big(\sigma_0, 1, (\frac{3\bar{s}^2}{4}, \frac{3\bar{s}^2}{4})\big)$. Indeed:

- This new policy does no worse conditional on contacting a single dealer, since, by Lemma 1, it ensures execution with probability one and achieves the cost lower bound $\hat{c}_1$. Note that the information about s communicated by this new policy is a garbling of that communicated by the original policy. However, there is no role for information design when only one dealer is contacted, because $\hat{c}_1$ is constant in ϕ for the reasons discussed at the end of Section 3.2.

- This new policy also does no worse conditional on contacting two dealers—and may in fact do strictly better. By Lemma 2, this policy ensures execution with probability one and achieves the cost $\hat{c}_2$ evaluated at the belief $\frac{\phi_0 q_{\bar{s}}}{(1-\phi_0)q_{-\bar{s}} + \phi_0 q_{\bar{s}}}$. As mentioned, the information about s communicated by this new policy is a garbling of that communicated by the original policy. Thus, by standard arguments from the Bayesian persuasion literature (e.g., Kamenica and Gentzkow, 2011), the new policy is guaranteed to reduce the client's cost if $\hat{c}_2(\phi)$ is convex, which by Lemma 2 is indeed the case.

It only remains to argue that such an RFQ policy achieves optimality. To see this, note that policies of this form are described by two numbers: $q_{-\bar{s}} \in [0, 1]$ and $q_{\bar{s}} \in [0, 1]$. Thus, the claim follows because (i) this class of policies is compact, and (ii) within this class of policies, the client's procurement cost is a continuous function of $(q_{-\bar{s}}, q_{\bar{s}})$. $\square$

A.4 Proof of Proposition 4

Proof. To begin, it is useful to observe that it follows from Definition 1 that $\underline{\phi} = 0$ implies $\hat{c}_2(\underline{\phi}) - \underline{\phi}\hat{c}_2'(\underline{\phi}) \leq \hat{c}_1$; $\underline{\phi} \in (0,1)$ implies $\hat{c}_2(\underline{\phi}) - \underline{\phi}\hat{c}_2'(\underline{\phi}) = \hat{c}_1$; and $\underline{\phi} = 1$ implies $\hat{c}_2(\underline{\phi}) - \underline{\phi}\hat{c}_2'(\underline{\phi}) \geq \hat{c}_1$.

As noted in the text, it suffices to focus on RFQ policies of the form described in Proposition 3. Such RFQ policies are described by two numbers: $q_{-\bar{s}}$ and $q_{\bar{s}}$, which capture the probability with which the client contacts two dealers when $s = -\bar{s}$ and $s = \bar{s}$, respectively. This implies that if two dealers are contacted, then the posterior probability of $s = \bar{s}$ is

$$\frac{\phi_0 q_{\bar{s}}}{(1-\phi_0)q_{-\bar{s}} + \phi_0 q_{\bar{s}}}.$$

Hence, the client's expected procurement cost is

$$[(1-\phi_0)(1-q_{-\bar{s}}) + \phi_0(1-q_{\bar{s}})]\,\hat{c}_1 + [(1-\phi_0)q_{-\bar{s}} + \phi_0 q_{\bar{s}}]\,\hat{c}_2\left(\frac{\phi_0 q_{\bar{s}}}{(1-\phi_0)q_{-\bar{s}} + \phi_0 q_{\bar{s}}}\right). \tag{30}$$

To prove claim *(i)*, we use $\hat{c}_1 \geq \min\{\hat{c}_1, \hat{c}_2(\phi)\} \geq C(\phi)$ and $\hat{c}_2(\phi) \geq \min\{\hat{c}_1, \hat{c}_2(\phi)\} \geq C(\phi)$ for all $\phi \in [0,1]$ to obtain that (30) is bounded below by

$$\begin{aligned}
&[(1-\phi_0)(1-q_{-\bar{s}}) + \phi_0(1-q_{\bar{s}})]\,C\left(\frac{\phi_0(1-q_{\bar{s}})}{(1-\phi_0)(1-q_{-\bar{s}}) + \phi_0(1-q_{\bar{s}})}\right)\\
&\quad + [(1-\phi_0)q_{-\bar{s}} + \phi_0 q_{\bar{s}}]\,C\left(\frac{\phi_0 q_{\bar{s}}}{(1-\phi_0)q_{-\bar{s}} + \phi_0 q_{\bar{s}}}\right),
\end{aligned}$$

which is bounded below by $C(\phi_0)$, by the convexity of $C(\cdot)$.

To prove claim *(iii)*, we begin by defining the following linear function

$$L_{(iii)}(\phi) = \hat{c}_2(\phi_0) - \phi_0\hat{c}_2'(\phi_0) + \hat{c}_2'(\phi_0)\phi.$$

We claim that for all $\phi \in [0,1]$, $L_{(iii)}(\phi) \leq \min\{\hat{c}_1, \hat{c}_2(\phi)\}$:

- First, we show that $L_{(iii)}(\phi) \leq \hat{c}_1$ for all $\phi \in [0,1]$. By linearity, it suffices to check the endpoints. By assumption, $\phi_0 \geq \underline{\phi}$, and since we also have $\phi_0 \in (0,1)$, this implies that $\underline{\phi} < 1$. Hence, as observed above, we have $\hat{c}_2(\underline{\phi}) - \underline{\phi}\hat{c}_2'(\underline{\phi}) \leq \hat{c}_1$. Additionally, as established in footnote 17, $\hat{c}_2(\phi) - \phi\hat{c}_2'(\phi)$ is strictly decreasing. Thus, because $\phi_0 \geq \underline{\phi}$, it follows that

$$L_{(iii)}(0) = \hat{c}_2(\phi_0) - \phi_0\hat{c}_2'(\phi_0) \leq \hat{c}_2(\underline{\phi}) - \underline{\phi}\hat{c}_2'(\underline{\phi}) \leq \hat{c}_1.$$

  An analogous argument establishes that $L_{(iii)}(1) \leq \hat{c}_1$.

- Second, note that $L_{(iii)}(\phi)$ is tangent to $\hat{c}_2(\phi)$ at $\phi = \phi_0$. Indeed, we have both $L_{(iii)}(\phi_0) = \hat{c}_2(\phi_0)$

and also $L'_{(iii)}(\phi_0) = \hat{c}_2'(\phi_0)$. Thus, convexity of $\hat{c}_2(\phi)$ implies that $L_{(iii)}(\phi) \leq \hat{c}_2(\phi)$ for all $\phi \in [0,1]$.

It follows that $C(\phi_0) \geq L_{(iii)}(\phi_0) = \hat{c}_2(\phi_0)$, which is precisely what is obtained from plugging $q_{-\bar{s}} = 1$ and $q_{\bar{s}} = 1$ into (30).

To prove claim *(iv)*, we begin by defining the following linear function

$$L_{(iv)}(\phi) = \hat{c}_1 + \frac{\hat{c}_2(\underline{\phi}) - \hat{c}_1}{\underline{\phi}}\phi.$$

We claim that for all $\phi \in [0,1]$, $L_{(iv)}(\phi) \leq \min\{\hat{c}_1, \hat{c}_2(\phi)\}$:

- First, we show that $L_{(iv)}(\phi) \leq \hat{c}_2(\phi)$ for all $\phi \in [0,1]$. By assumption, $\phi_0 < \underline{\phi}$, which implies that $\underline{\phi} > 0$. On the one hand, if $\underline{\phi} \in (0,1)$, then $L_{(iv)}(\phi)$ is tangent to $\hat{c}_2(\phi)$ at $\phi = \underline{\phi}$. Indeed, we have both $L_{(iv)}(\underline{\phi}) = \hat{c}_2(\underline{\phi})$ and also $L'_{(iv)}(\underline{\phi}) = \frac{\hat{c}_2(\underline{\phi})-\hat{c}_1}{\underline{\phi}} = \hat{c}_2'(\underline{\phi})$, where the last equality is because, as observed above, $\underline{\phi} \in (0,1)$ implies $\hat{c}_2(\underline{\phi}) - \underline{\phi}\hat{c}_2'(\underline{\phi}) = \hat{c}_1$. On the other hand, if $\underline{\phi} = 1$, then by analogous arguments we have both $L_{(iv)}(\underline{\phi}) = \hat{c}_2(\underline{\phi})$ and $L'_{(iv)}(\underline{\phi}) \leq \hat{c}_2'(\underline{\phi})$. In either case, convexity of $\hat{c}_2(\phi)$ implies that $L_{(iv)}(\phi) \leq \hat{c}_2(\phi)$ for all $\phi \in [0,1]$.

- Second, we show that $L_{(iv)}(\phi) \leq \hat{c}_1$ for all $\phi \in [0,1]$. We showed above that $L_{(iv)}(\phi) \leq \hat{c}_2(\phi)$ for all $\phi \in [0,1]$; so in particular, we have $L_{(iv)}(\frac{1}{2}) \leq \hat{c}_2(\frac{1}{2})$. According to Lemma 2, $\hat{c}_2(\frac{1}{2}) < \hat{c}_1$. Hence, $L_{(iv)}(\frac{1}{2}) < \hat{c}_1$, which implies $\hat{c}_2(\underline{\phi}) - \hat{c}_1 < 0$, and thus $L_{(iv)}(\phi) \leq \hat{c}_1$ for all $\phi \in [0,1]$, as desired.

It follows that

$$C(\phi_0) \geq L_{(iv)}(\phi_0) = \frac{\hat{c}_2(\underline{\phi_0}) - \hat{c}_1}{\underline{\phi}}\phi_0 + \hat{c}_1,$$

which is precisely what is obtained from plugging $q_{-\bar{s}} = \frac{\phi_0(1-\underline{\phi})}{\underline{\phi}(1-\phi_0)}$ and $q_{\bar{s}} = 1$ into (30). The proof of claim *(v)* is analogous.

Finally, we prove claim *(ii)*: that $\underline{\phi} \leq \overline{\phi}$. The claim is non-vacuous only if $\underline{\phi} > 0$. So as established in the proof of claim *(iv)*, $\hat{c}_2(\underline{\phi}) - \hat{c}_1 < 0$. And as observed above, $\underline{\phi} > 0$ implies $\hat{c}_2(\underline{\phi}) - \phi\hat{c}_2'(\underline{\phi}) \geq \hat{c}_1$. Together, these imply that $\hat{c}_2'(\underline{\phi}) < 0$. Analogously, the claim is non-vacuous only if $\overline{\phi} < 1$, and we can analogously argue that $\hat{c}_2'(\overline{\phi}) > 0$. Convexity of $\hat{c}_2(\cdot)$ therefore delivers the desired $\underline{\phi} \leq \overline{\phi}$. □

A.5 Proof of Lemma 5

Proof. A derivation analogous to that in the proof of Lemma 2 allows us to characterize the client's expected procurement cost conditional on her type:

$$\begin{aligned}
\hat{c}_{2,-\bar{s}}(\phi) = & \frac{(2304 - 560\phi - 157\phi^2)\bar{s}^2}{4(24-\phi)^2}\psi[1-(1-\psi)(1-\rho)] \\
& - \frac{207\phi^2\bar{s}^2}{4(23+\phi)^2}(1-\psi)[1-\psi(1-\rho)] + \frac{7\bar{s}^2}{16}2\psi(1-\psi)(1-\rho) \\
\hat{c}_{2,\bar{s}}(\phi) = & \frac{(1587 + 874\phi - 157\phi^2)\bar{s}^2}{4(23+\phi)^2}(1-\psi)[1-\psi(1-\rho)] \\
& - \frac{207(1-\phi)^2\bar{s}^2}{4(24-\phi)^2}\psi[1-(1-\psi)(1-\rho)] + \frac{7\bar{s}^2}{16}2\psi(1-\psi)(1-\rho).
\end{aligned}$$

To establish the claim, we simply compute

$$\hat{c}'_{2,-\bar{s}}(\phi) = -\frac{184(12+11\phi)\bar{s}^2}{(24-\phi)^3}\psi[1-(1-\psi)(1-\rho)] - \frac{4761\phi\bar{s}^2}{2(23+\phi)^3}(1-\psi)[1-\psi(1-\rho)],$$

which is indeed weakly negative on the domain $\phi \in [0,1]$.

$$\hat{c}'_{2,\bar{s}}(\phi) = \frac{184(23-11\phi)\bar{s}^2}{(23+\phi)^3}(1-\psi)[1-\psi(1-\rho)] + \frac{4761(1-\phi)\bar{s}^2}{2(24-\phi)^3}\psi[1-(1-\psi)(1-\rho)],$$

which is indeed weakly positive on the domain $\phi \in [0,1]$. □

B Order splitting

In this appendix, we consider what would happen if the client were to contact both dealers and—rather than auction her entire order as a single indivisible unit—instead permits each dealer to win only half of the total order. For simplicity, suppose the client continues to treat the order as indivisible in the sense that she allocates the order only if both dealers' bids meet her reservation price. In other words, the game ends if one or both dealers fail to meet the reserve.

Given the structure of the model, it is common knowledge that the client seeks to trade $\bar{s}$ shares. Thus, if a dealer is awarded an order for $\frac{\bar{s}}{2}$ shares, he knows that the other dealer is also being awarded an order of the same size. Is this knowledge an artifact of the model or is it realistic? Based on conversations with industry participants, we believe the latter. Although in principle, it would be possible to deceive two dealers into believing that each has been awarded the full order, when in reality each has been awarded only a fraction, this behavior is avoided in practice because of the reputational consequences it would create.

The following result characterizes the continuation equilibrium following such an RFQ.

Lemma B1. *There is a WPBE in which the following occurs on path. Dealer A bids*

$$\left(b^A_{-\bar{s}}, b^A_{\bar{s}}\right) = \begin{cases} \left(\frac{7\bar{s}^2}{18}, 0\right) & \text{if } (e^A, e^B) = (1,1) \\ \left(\frac{7\bar{s}^2}{50}, -\frac{7\bar{s}^2}{100}\right) & \text{if } (e^A, e^B) = (1,-1) \\ \left(-\frac{7\bar{s}^2}{100}, \frac{7\bar{s}^2}{50}\right) & \text{if } (e^A, e^B) = (-1,1) \\ \left(0, \frac{7\bar{s}^2}{18}\right) & \text{if } (e^A, e^B) = (-1,-1) \end{cases}$$

If both dealers win, dealer A's on-market trades are

$$(x^A_1, x^A_2) = \begin{cases} (0,0) & \text{if } (s, e^A, e^B) = (\bar{s}, 1, 1) \\ (-\frac{\bar{s}}{3}, -\frac{\bar{s}}{6}) & \text{if } (s, e^A, e^B) = (-\bar{s}, 1, 1) \\ (\frac{\bar{s}}{10}, -\frac{3\bar{s}}{10}) & \text{if } (s, e^A, e^B) = (\bar{s}, 1, -1) \\ (-\frac{\bar{s}}{10}, -\frac{2\bar{s}}{5}) & \text{if } (s, e^A, e^B) = (-\bar{s}, 1, -1) \\ (\frac{\bar{s}}{10}, \frac{2\bar{s}}{5}) & \text{if } (s, e^A, e^B) = (\bar{s}, -1, 1) \\ (-\frac{\bar{s}}{10}, \frac{3\bar{s}}{10}) & \text{if } (s, e^A, e^B) = (-\bar{s}, -1, 1) \\ (\frac{\bar{s}}{3}, \frac{\bar{s}}{6}) & \text{if } (s, e^A, e^B) = (\bar{s}, -1, -1) \\ (0,0) & \text{if } (s, e^A, e^B) = (-\bar{s}, -1, -1) \end{cases}$$

Dealer B's bids and on-market trades are specified symmetrically.

Proof Sketch of Lemma B1. The proof sketch that we provide here is informal in that we (*i*) directly plug in the constraints that will bind on the equilibrium path, and (*ii*) ignore the constraints that do not bind on the equilibrium path. These simplifications do not affect the result. Indeed, the proof could be made more formal in the same way that the proofs of the analogous main results (Lemmas 1 and 2) are fully formal.

Because both dealers observe the entire vector (e^A, e^B), the four possible realizations of that vector can be analyzed separately. Below, we analyze the cases of $(1,1)$ and $(1,-1)$; the remaining cases can be handled symmetrically. We also note that in this case of order splitting, both dealers observe s directly before trading takes place, and so there is no need to keep track of beliefs. Thus, the cases of $s=-\bar{s}$ and $s=\bar{s}$ can also be analyzed separately.

Case 1: $(e^A, e^B) = (1,1)$ and $s=\bar{s}$. Ignoring the constraints on final inventory (which will not bind in the equilibrium), dealers A and B respectively minimize

$$(x_1^A + x_1^B)x_1^A + (x_1^A + x_1^B + x_2^A + x_2^B)x_2^A$$
$$(x_1^A + x_1^B)x_1^B + (x_1^A + x_1^B + x_2^A + x_2^B)x_2^B,$$

leading to $x_2^A = x_2^B = -\frac{x_1^A + x_1^B}{3}$. Inducting backward, we obtain $(x_1^A, x_1^B) = (0,0)$, so that $(x_2^A, x_2^B) = (0,0)$ on path. Plugging in these trades, dealer A incurs no trading costs if he wins. So the refinement described in Section 3.1 requires $b_{\bar{s}}^A = 0$ to be his bid.

Case 2: $(e^A, e^B) = (1,1)$ and $s = -\bar{s}$. Assuming that $x_2^A = -\frac{\bar{s}}{2} - x_1^A$ (which ensures that dealer A's final inventory just meets the constraint $e^A + x_1^A + x_2^A - s \leq 1$), assuming also that $x_2^B = -\frac{\bar{s}}{2} - x_1^B$ (symmetrically), and ignoring all other constraints on final inventory, dealers A and B respectively minimize

$$(x_1^A + x_1^B)x_1^A + (-\bar{s})\left(-\frac{\bar{s}}{2} - x_1^A\right)$$
$$(x_1^A + x_1^B)x_1^B + (-\bar{s})\left(-\frac{\bar{s}}{2} - x_1^B\right),$$

leading to $(x_1^A, x_1^B) = (-\frac{\bar{s}}{3}, -\frac{\bar{s}}{3})$, which implies $(x_2^A, x_2^B) = (-\frac{\bar{s}}{6}, -\frac{\bar{s}}{6})$. Plugging in these trades, dealer A incurs trading costs of $\frac{7\bar{s}^2}{18}$ if he wins. So the refinement described in Section 3.1 requires $b_{-\bar{s}}^A = \frac{7\bar{s}^2}{18}$ to be his bid.

Case 3: $(e^A, e^B) = (1,-1)$ and $s = \bar{s}$. Assuming that $x_2^B = \frac{\bar{s}}{2} - x_1^B$ (which ensures that dealer B's final inventory just meets the constraint $e^B + x_1^B + x_2^B - s \geq -1$), and ignoring all other constraints on final inventory, dealers A and B respectively minimize

$$(x_1^A + x_1^B)x_1^A + \left(x_1^A + x_2^A + \frac{\bar{s}}{2}\right)x_2^A$$
$$(x_1^A + x_1^B)x_1^B + \left(x_1^A + x_2^A + \frac{\bar{s}}{2}\right)\left(\frac{\bar{s}}{2} - x_1^B\right),$$

leading to $x_2^A = -\frac{x_1^A}{2} - \frac{\bar{s}}{4}$. Inducting backward, we obtain $(x_1^A, x_1^B) = \left(\frac{\bar{s}}{10}, \frac{\bar{s}}{10}\right)$, so that $(x_2^A, x_2^B) = \left(-\frac{3\bar{s}}{10}, \frac{2\bar{s}}{5}\right)$ on path. Plugging in these trades, dealer A incurs trading costs of $-\frac{7\bar{s}^2}{100}$ if he wins. So the refinement described in Section 3.1 requires $b_{\bar{s}}^A = \frac{7\bar{s}^2}{100}$ to be his bid.

Case 4: $(e^A, e^B) = (1,-1)$ and $s = -\bar{s}$. Assuming that $x_2^A = -\frac{\bar{s}}{2} - x_1^A$ (which ensures that dealer A's final inventory just meets the constraint $e^A + x_1^A + x_2^A - s \leq 1$), and ignoring all other constraints on final inventory, dealers A and B respectively minimize

$$(x_1^A + x_1^B)x_1^A + \left(x_1^B + x_2^B - \frac{\bar{s}}{2}\right)\left(-\frac{\bar{s}}{2} - x_1^A\right)$$
$$(x_1^A + x_1^B)x_1^B + \left(x_1^B + x_2^B - \frac{\bar{s}}{2}\right)x_2^B,$$

leading to $x_2^B = -\frac{x_1^B}{2} + \frac{\bar{s}}{4}$. Inducting backward, we obtain $(x_1^A, x_1^B) = \left(-\frac{\bar{s}}{10}, -\frac{\bar{s}}{10}\right)$, so that $(x_2^A, x_2^B) = \left(-\frac{2\bar{s}}{5}, \frac{3\bar{s}}{10}\right)$ on path. Plugging in these trades, dealer A incurs trading costs of $\frac{7\bar{s}^2}{50}$ if he wins. So the refinement described in Section 3.1 requires $b^A_{-\bar{s}} = \frac{7\bar{s}^2}{50}$ to be his bid. □

To ensure execution with probability one, Lemma B1 implies that an RFQ that splits the order in this way must entail reserve prices $\bar{b}_{-\bar{s}} \geq \frac{7\bar{s}^2}{18}$ and $\bar{b}_{\bar{s}} \geq \frac{7\bar{s}^2}{18}$. Hence, the minimum procurement cost that can be achieved by such an RFQ is twice this lower bound:

$$\hat{c}_{split} \equiv \frac{7\bar{s}^2}{9}.$$

Finally, observe that $\hat{c}_{split} > \hat{c}_1$. The intuition is that—just as when one dealer is contacted—the client's procurement cost is driven by the worst case. As before, there are two symmetric worst cases. But to fix ideas, focus on the one in which the client wants to buy while both dealers are initially short (i.e., $s = \bar{s}$ and $e^A = e^B = -1$). In this worst case:

- If one dealer is contacted, then he buys a total of $\bar{s}$ on the market. In doing so, he trades at an even rate, buying $\frac{\bar{s}}{2}$ in each of the two periods, which is the cost-minimizing way to trade under permanent price impact (e.g., Bertsimas and Lo, 1998).

- On the other hand, if the order is split among the two dealers, then each buys a total of $\frac{\bar{s}}{2}$ on the market. However, each front-loads their trading, buying $\frac{\bar{s}}{3}$ in the first period and $\frac{\bar{s}}{6}$ in the second. Thus, the dealers do not collectively act to minimize their aggregate trading cost. This increase in cost is ultimately passed on to the client.

  Why do the dealers not collectively act to minimize their aggregate trading cost? If an individual dealer shifts some volume from the second period into the first period, that raises p_1 but has no effect on p_2 (because of the permanent price impact). This affects that dealer's trading costs in two ways: (*i*) it reduces the trading cost for those marginal shares (because of the permanent price impact, $p_2 > p_1$), but (*ii*) it increases the trading cost for the inframarginal shares that were already traded in the first period. In addition, there is a negative externality on the first-period trading costs of the other dealer. But because this externality is not internalized, each dealer frontloads his trading.

  By splitting the order, the client loses the coordination benefits that she would obtain by allocating the entire order to a single dealer and instead "competes against herself."

Because $\hat{c}_{split} > \hat{c}_1$, the client optimally never splits her order in this way. Any RFQ policy that does entail such order splitting could be improved by contacting only one dealer and offering the entire order to him whenever the policy would call for order splitting. Thus, it was without loss of generality that we did not allow for such order splitting in our baseline analysis.

C Illustration of optimal policy under limited commitment power

Section 4 considered an alternative version of the model, in which the client cannot commit to randomization over how many dealers to contact. The optimal such policy was characterized by Proposition $4'$. This appendix provides an example that both illustrates and provides a geometric interpretation for that result.

In fact, we use the same parametrization that was used to produce Figure 1: $\psi = 0.85$ and $\rho = 1$. Then $\hat{c}_1$ and $\hat{c}_2(\phi)$ are as depicted in the first panel of Figure $1'$, which coincides with the first panel of Figure 1.

The second panel of Figure $1'$ depicts $\min\{\phi\hat{c}_2(1) + (1-\phi)\hat{c}_1, \phi\hat{c}_1 + (1-\phi)\hat{c}_2(0), \hat{c}_2(\phi_0)\}$, which we label $\tilde{C}(\phi)$. In this example, $\tilde{C}(\phi)$ is simply the lower envelope of $c_2(\phi)$ and the line connecting $(0, \hat{c}_1)$ to $\big(1, \hat{c}_2(1)\big)$. This second panel also depicts $\underset{\sim}{\phi}$, which is defined as the intersection between $c_2(\phi)$ and the aforementioned line. Alternatively, this is the minimum value for which $C(\phi) = \hat{c}_2(\phi)$. We also have $\tilde{\phi} = 1$ in this case, but we do not depict this in the figure because $\tilde{\phi}$ plays no role what follows.

The third panel of the figure relates to case *(iii)* of Proposition $4'$. Here, we have $\phi_0 \in [\underset{\sim}{\phi}, \tilde{\phi}]$. The optimal RFQ policy always contacts two dealers and discloses no information about the client's order. Under this policy, dealers' beliefs therefore always coincide with the prior, so that the client's expected cost is $\hat{c}_2(\phi_0)$.

Finally, the fourth panel of the figure relates to case *(iv)* of Proposition $4'$. Here, we have $\phi_0 \in (0, \underset{\sim}{\phi})$. The optimal RFQ policy always contacts two (one) dealers when $s = \bar{s}$ ($s = -\bar{s}$). Hence, if two (one) dealers are contacted, they believe $s = \bar{s}$ with probability one (zero). Under this policy, the client's expected procurement cost is therefore an appropriate convex combination of $\hat{c}_1$ and $\hat{c}_2(1)$, which is precisely what $\tilde{C}(\phi_0)$ captures.

Figure 1′: Optimal RFQ Policies ($\psi = 0.85$ and $\rho = 1$)